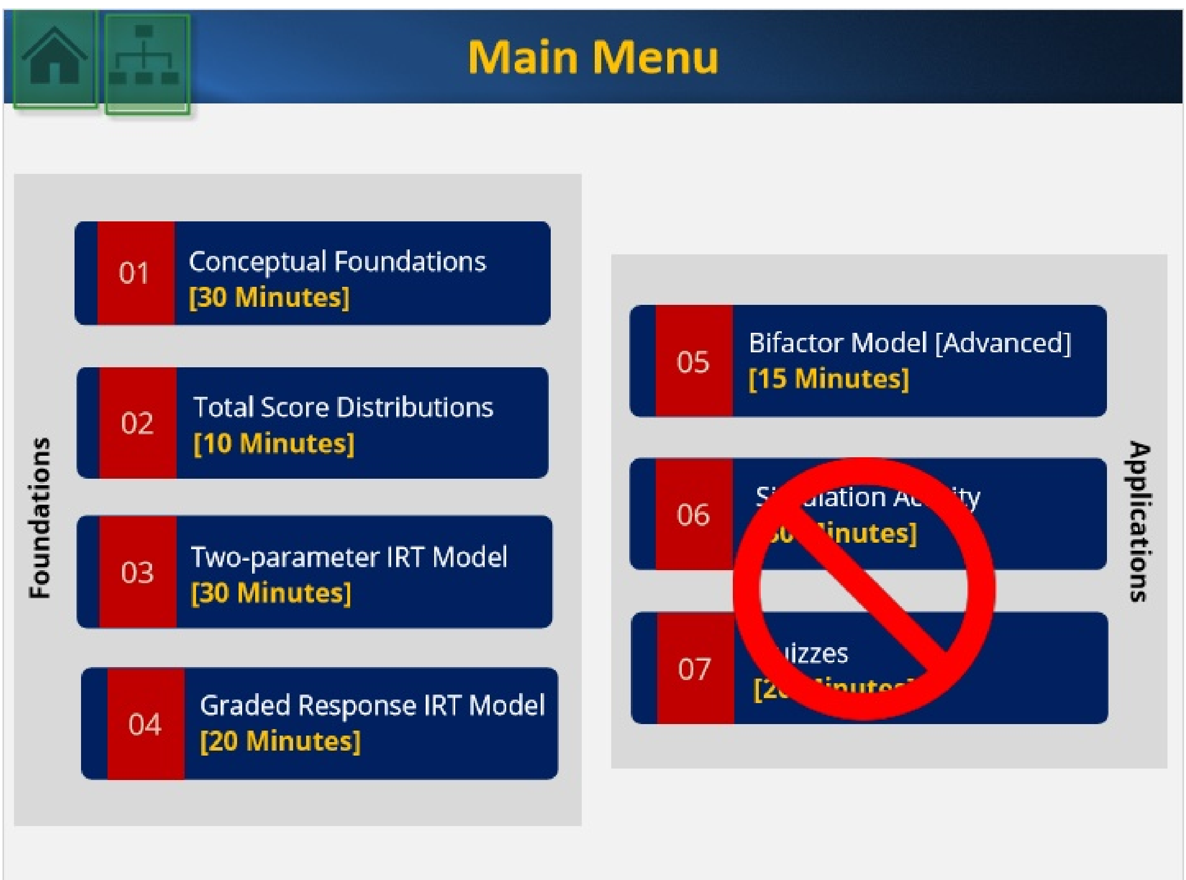
ITEMS DM13: Simulation Studies in Item Response Theory

This document contains only the core content slides from the module. In the digital module all slides can be accessed individually.

Module Organization

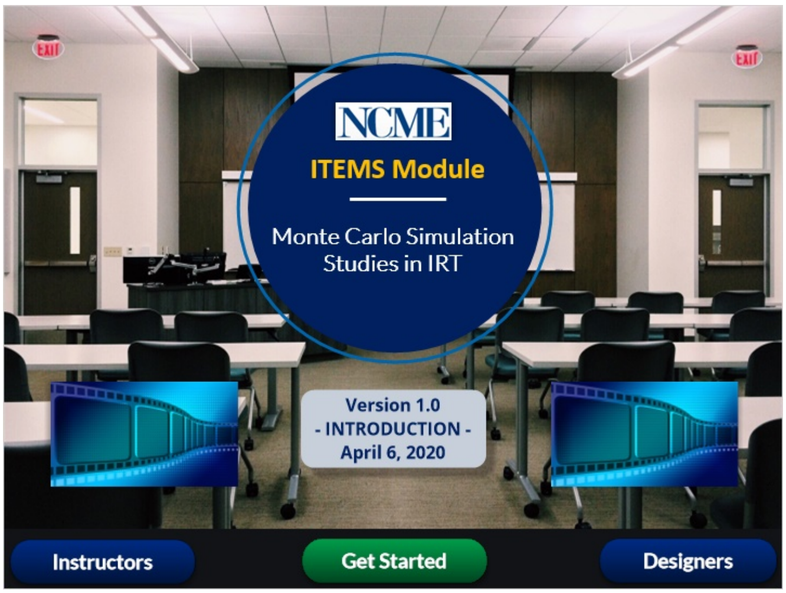
The module starts with an introductory section that leads to the main menu from which learners can select individual sections; this slide deck contains the slides for the five content sections only as shown below; green images in this document are clickable hotspot regions:



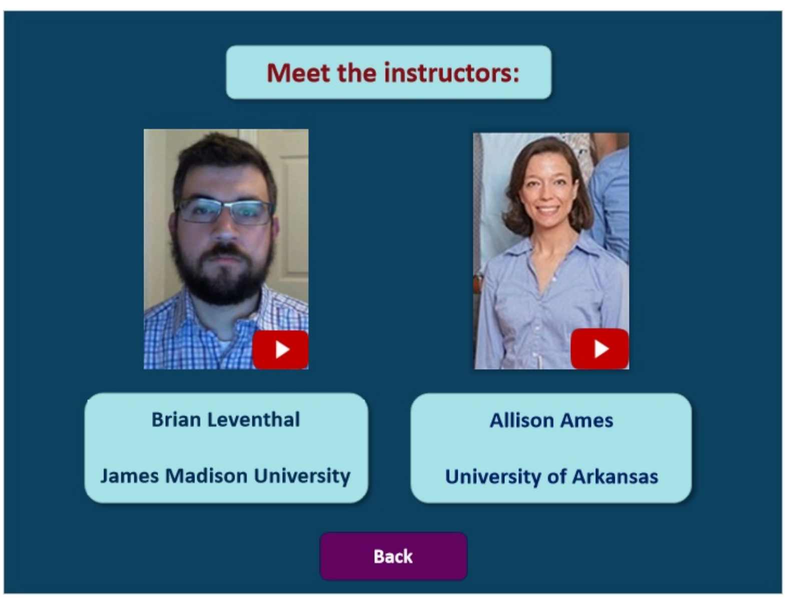
DM13 VIDEO (Section 5, Version 1.0)

1. Module Overview

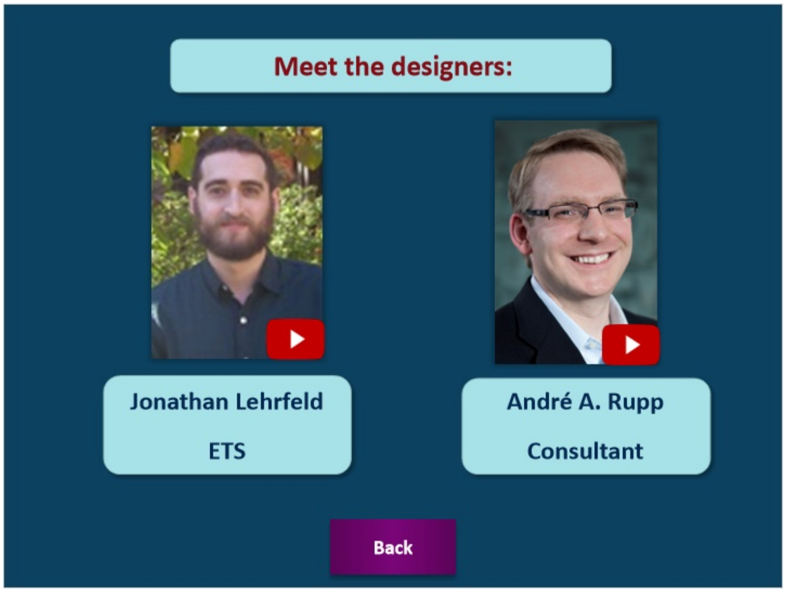
1.1 Module Cover (START)



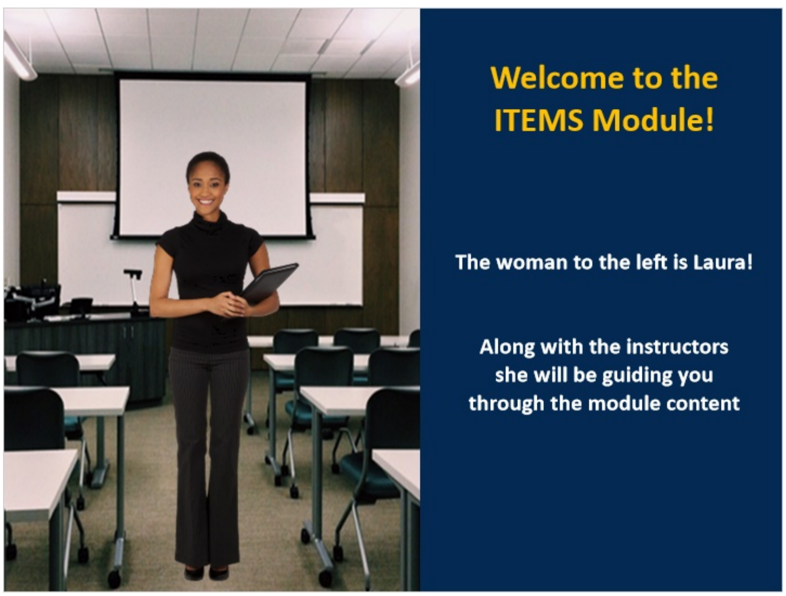
1.2 Content Team



1.3 Design Team

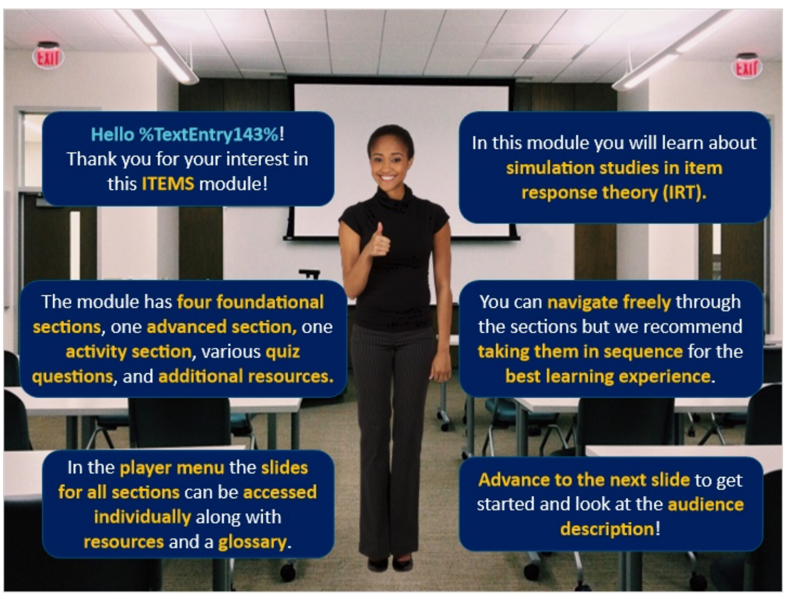


1.4 Welcome

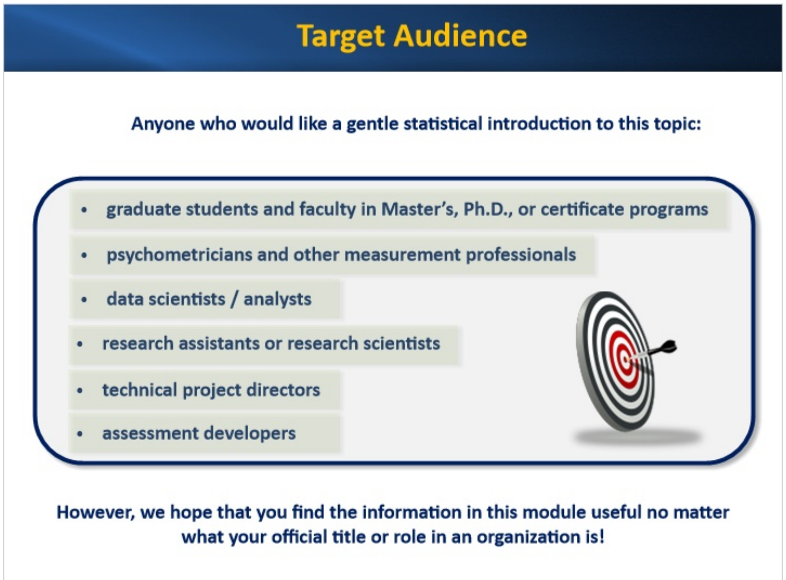


Untitled Layer 1 (Slide Layer)

1.5 Overview

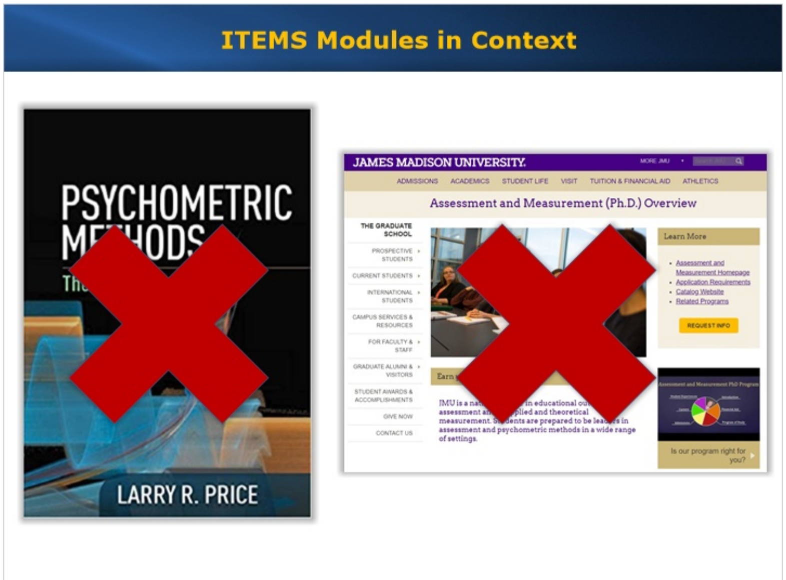


1.6 Target Audience

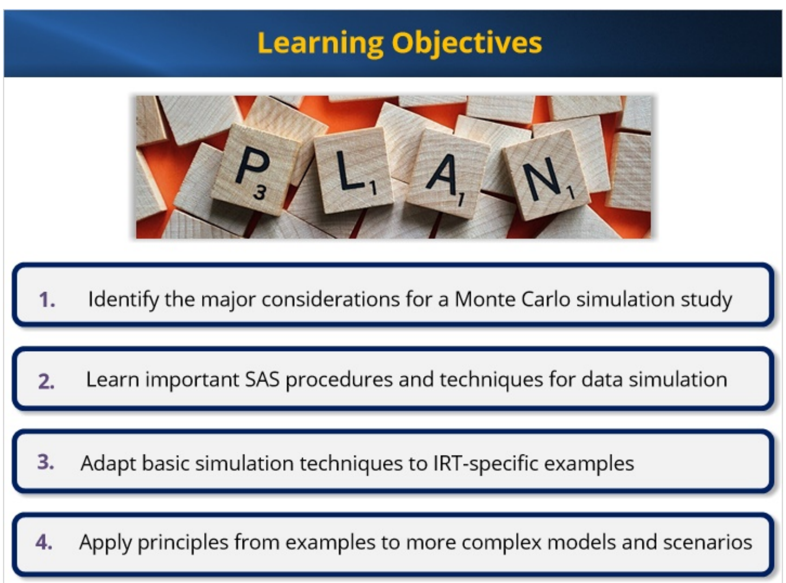


1.7 Expecations (I)

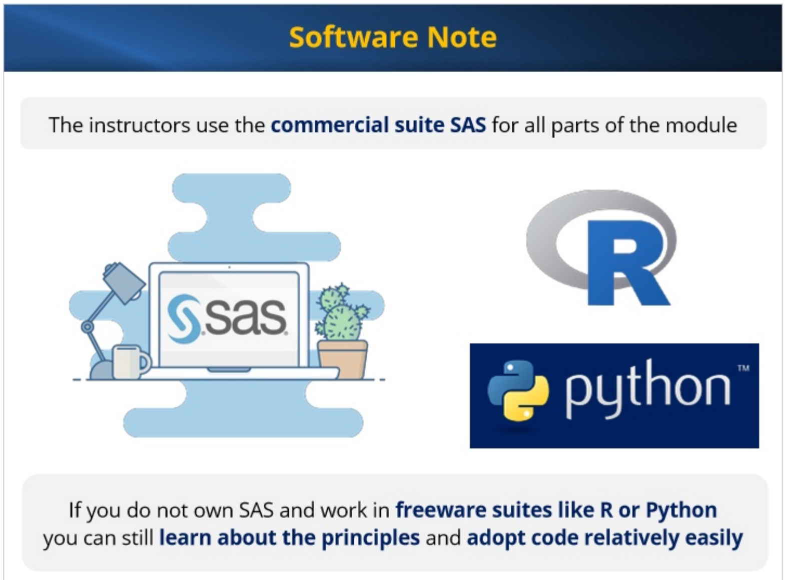
1.8 Expectations (II)



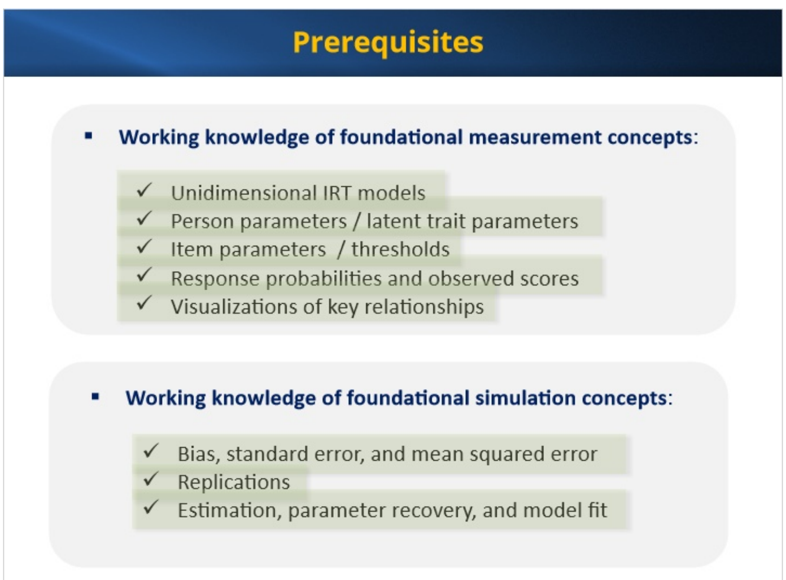
1.9 Learning Objectives



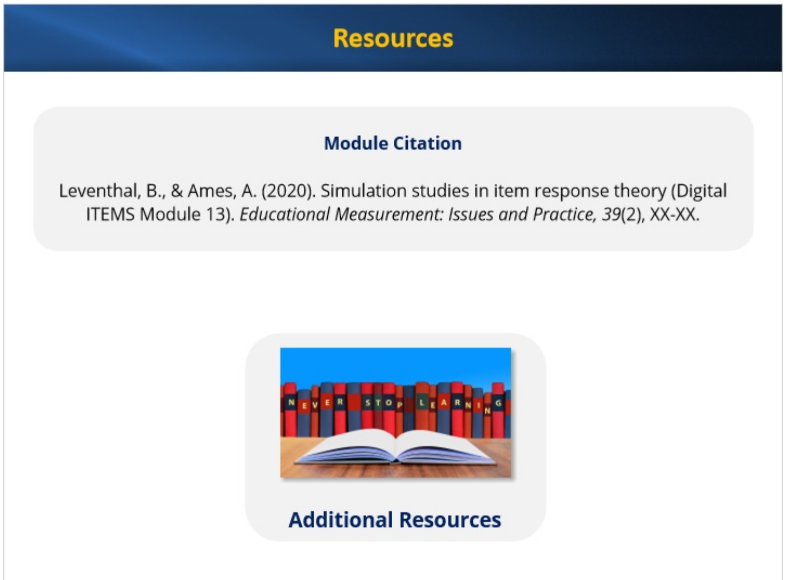
1.10 Software Note



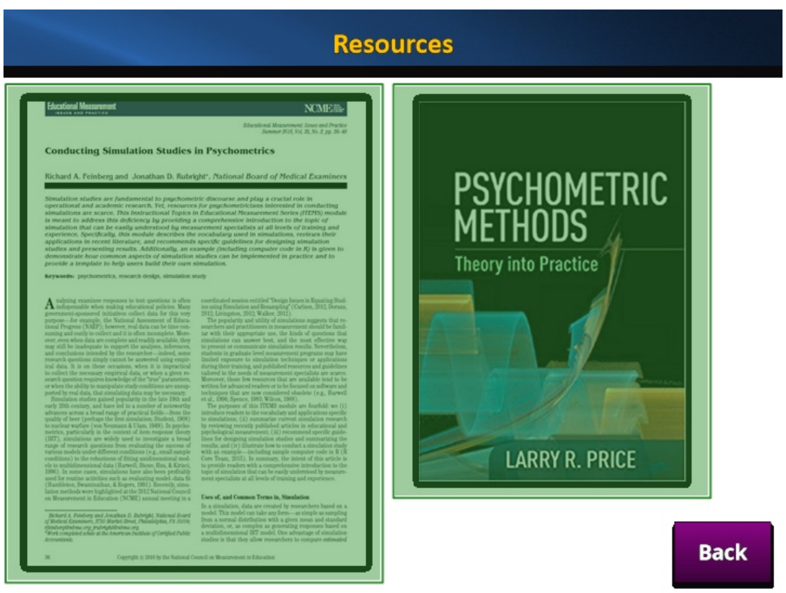
1.11 Prerequisites



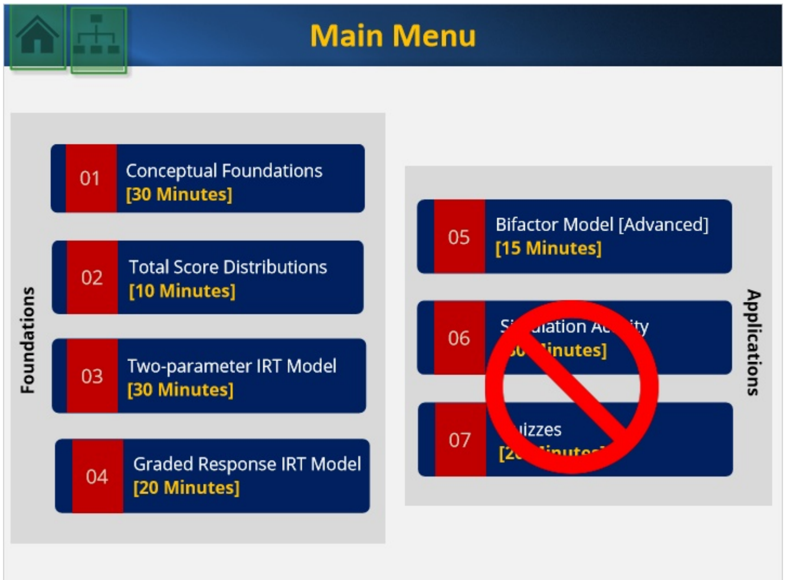
1.12 Resources



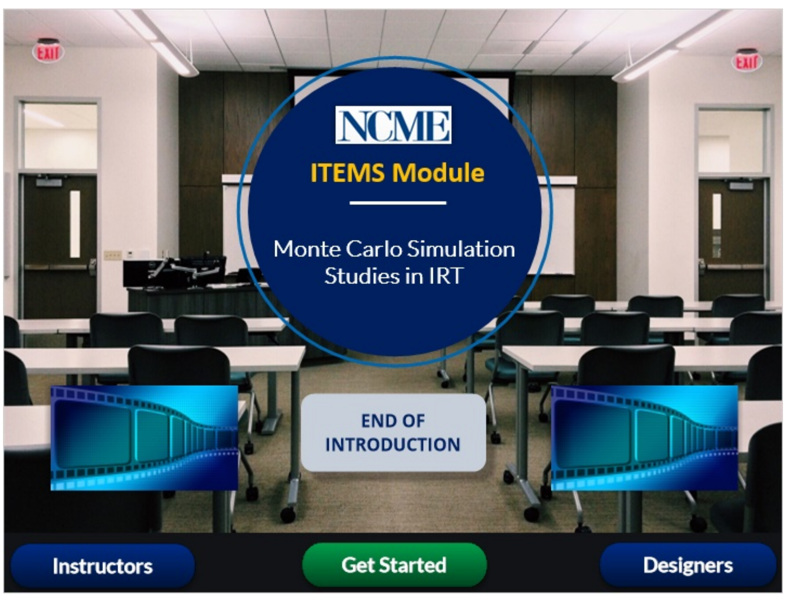
References (Slide Layer)



1.13 Main Menu

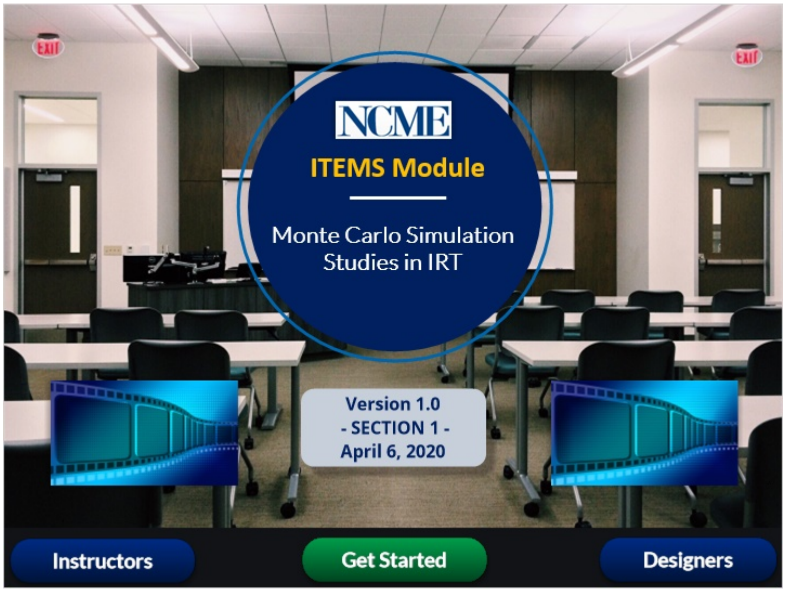


1.14 Module Cover (END)



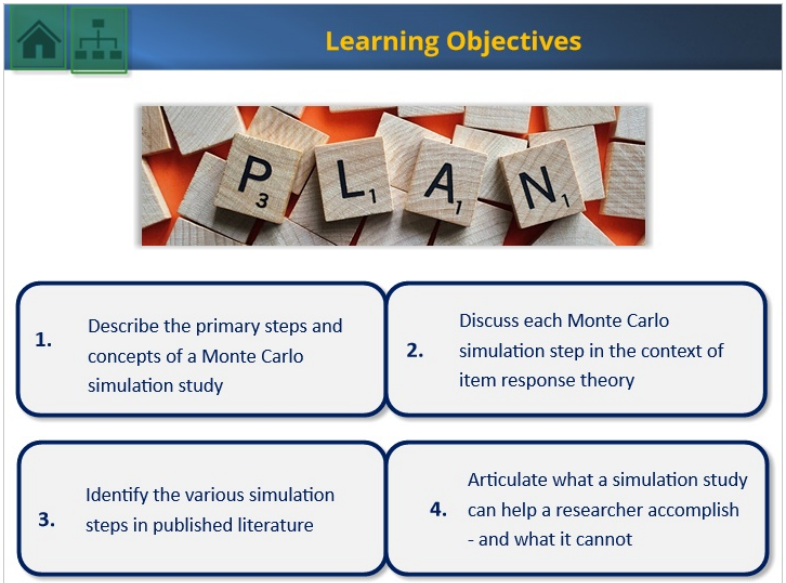
2. Section 1: Conceptual Foundations

2.1 Module Cover (START)

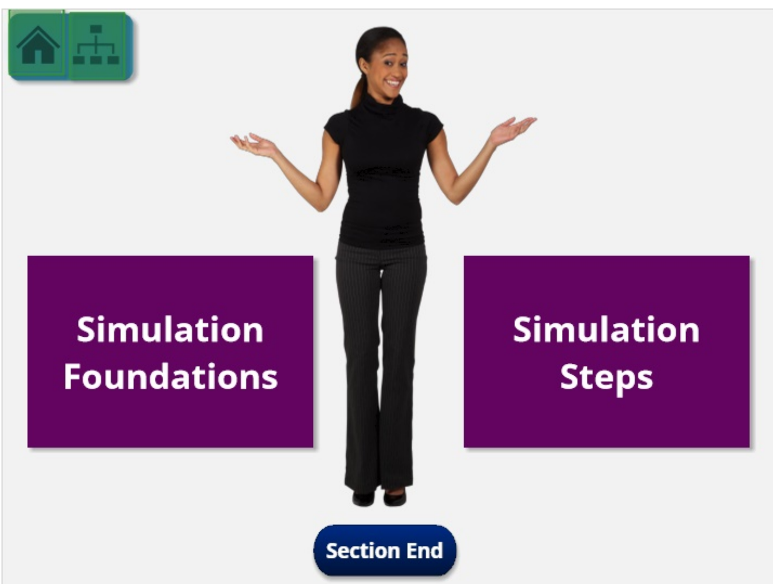


2.2 Cover: Section 1

2.3 Objectives

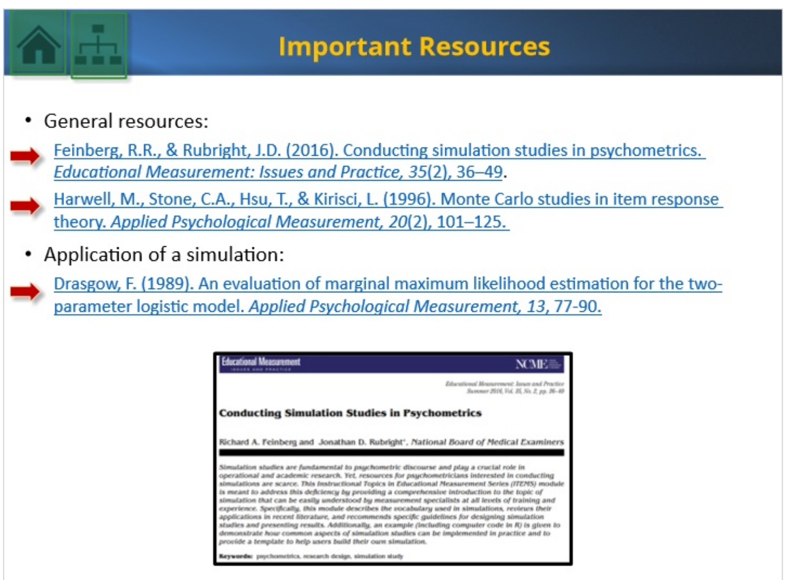


2.4 Topic Selection

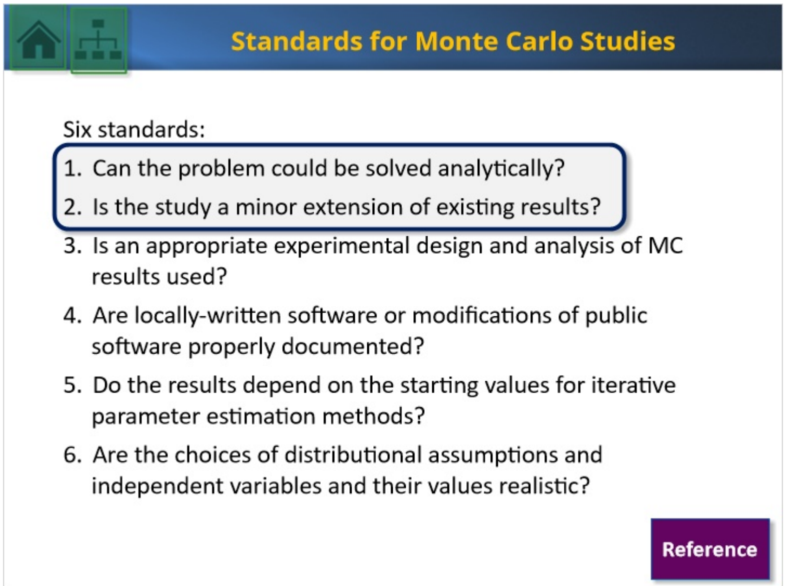


2.5 Bookmark: Foundations

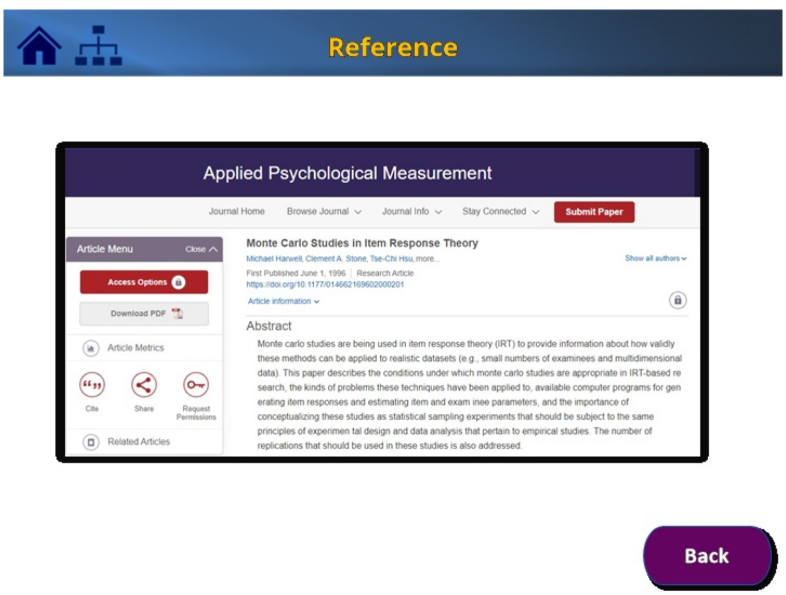
2.6 Important Resources



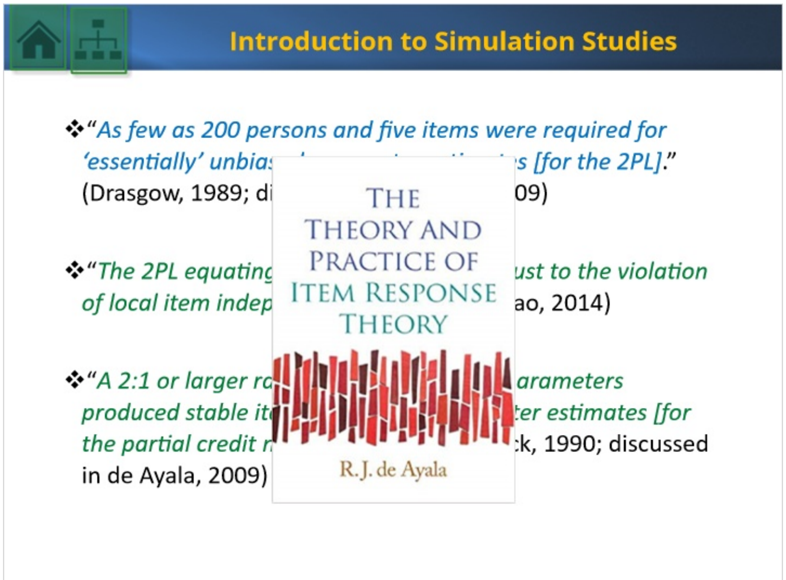
2.7 Standards



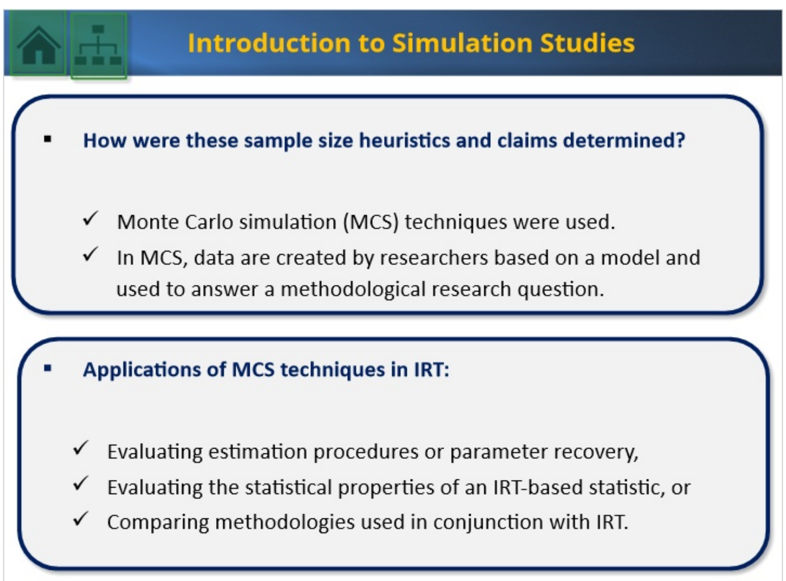
Reference (Slide Layer)



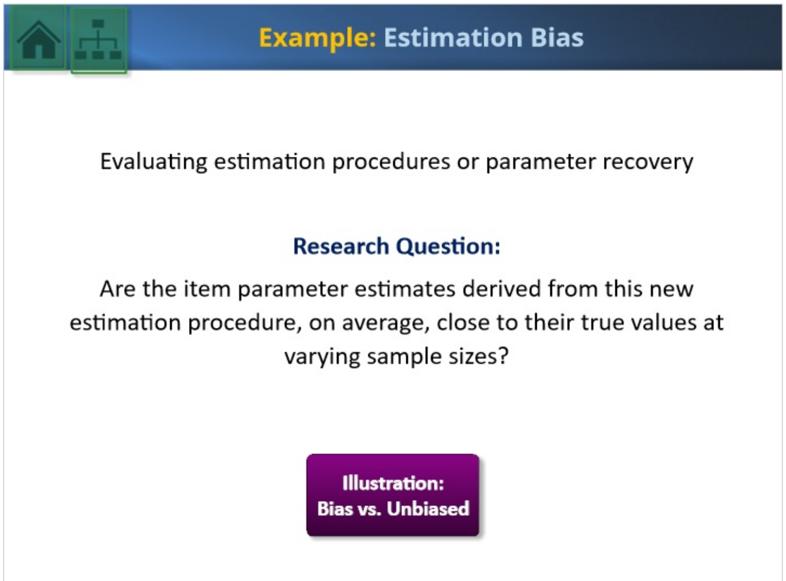
2.8 Introduction (I)



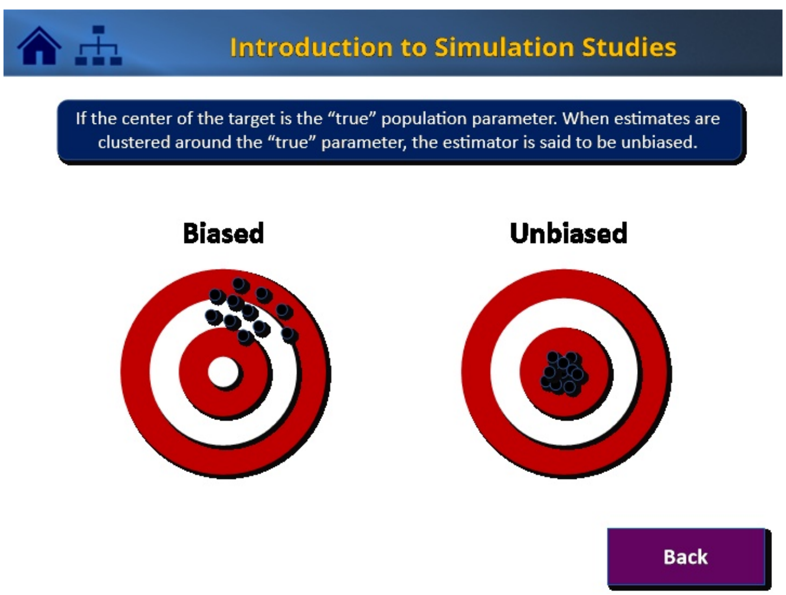
2.9 Introduction (II)



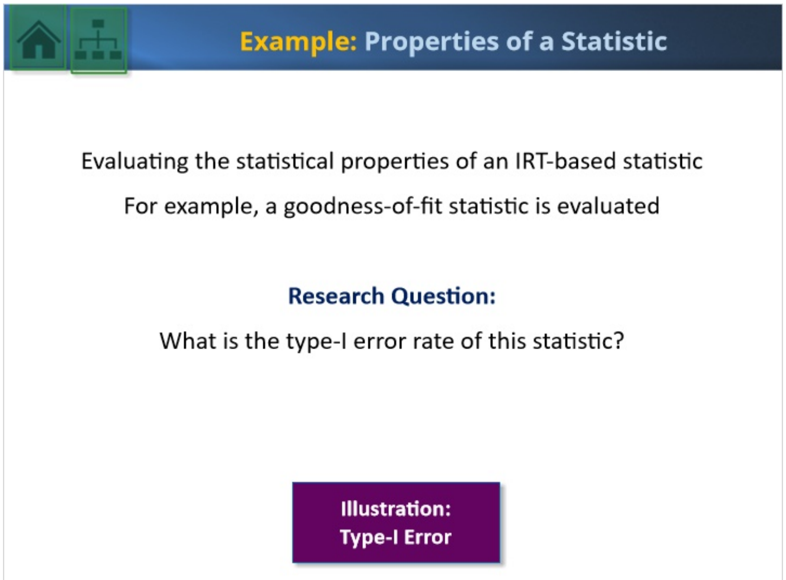
2.10 Example: Bias



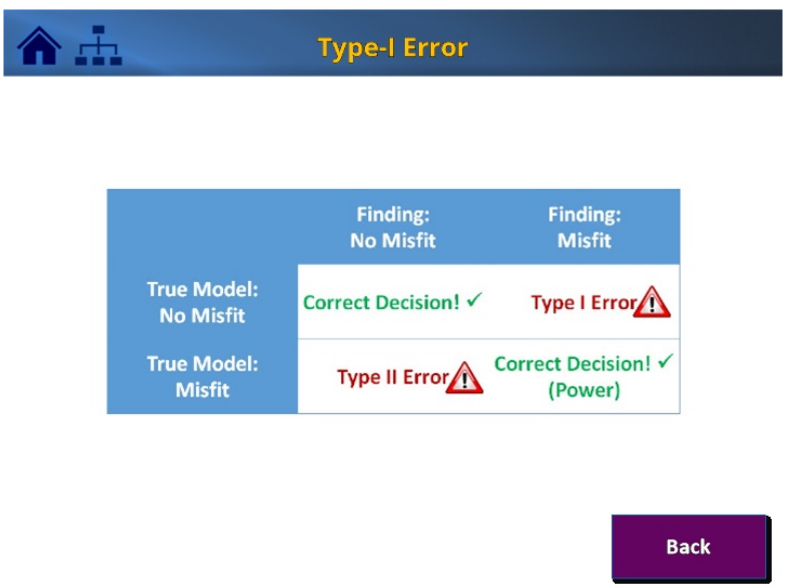
Introduction to Simulation Studies (Slide Layer)



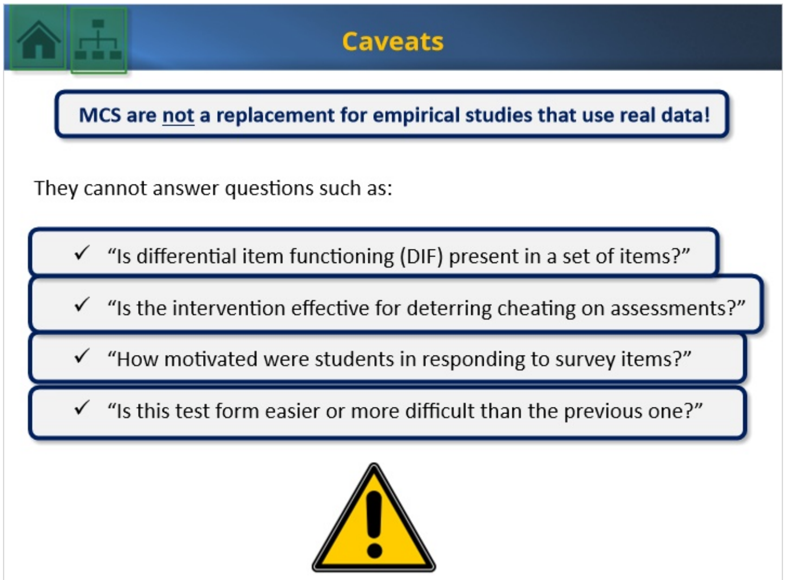
2.11 Example: Type-I Error



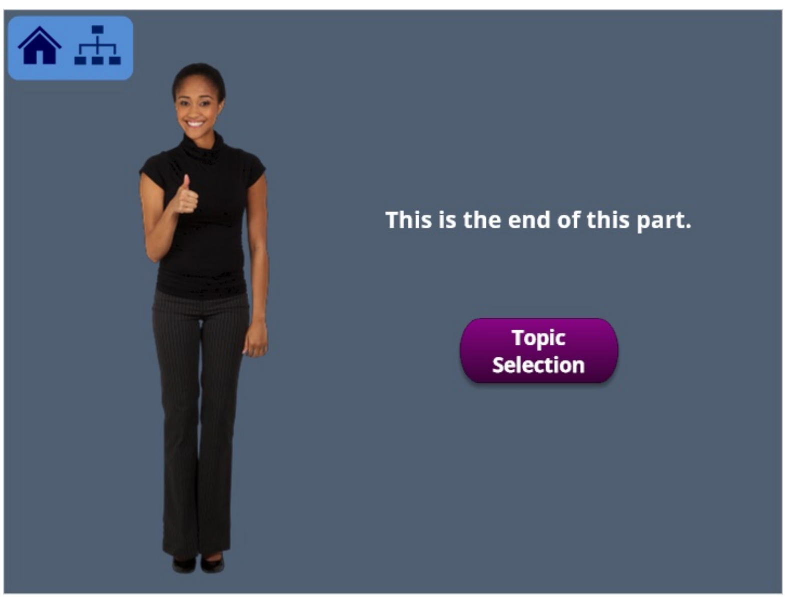
Type-I Error (Slide Layer)



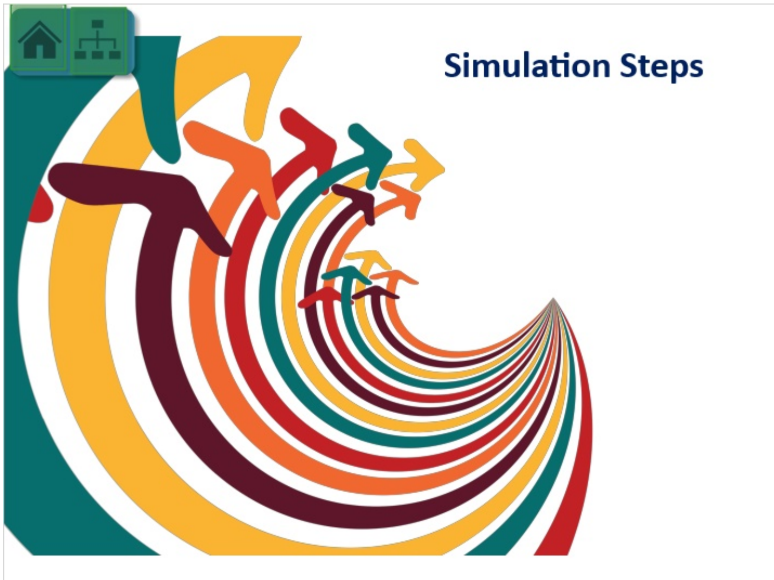
2.12 Caveats



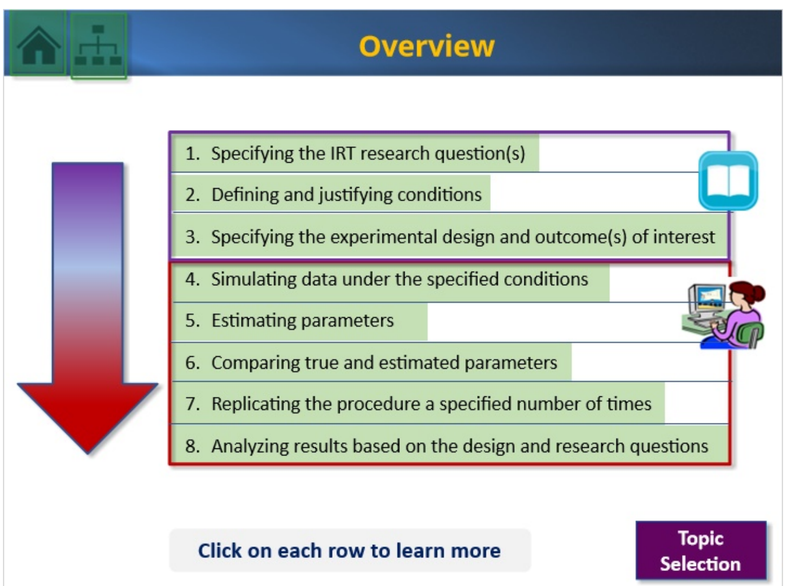
2.13 Bookend: Foundations



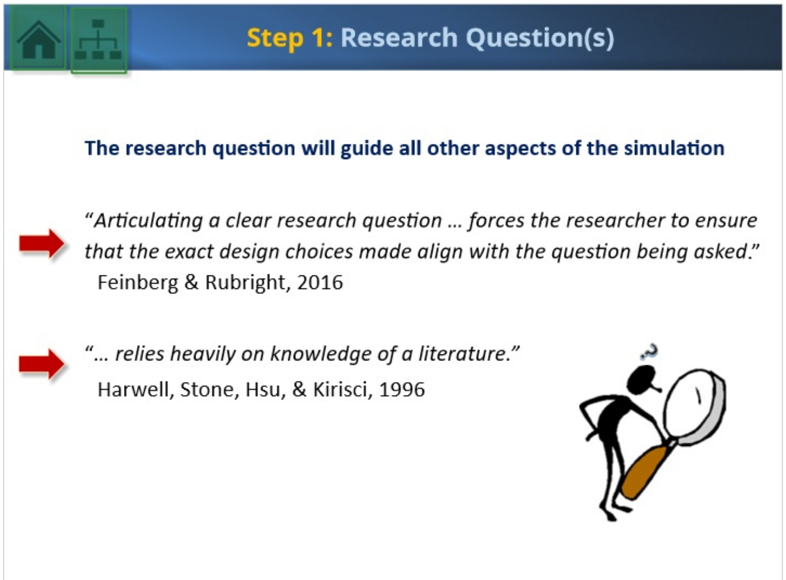
2.14 Bookmark: Simulation Steps



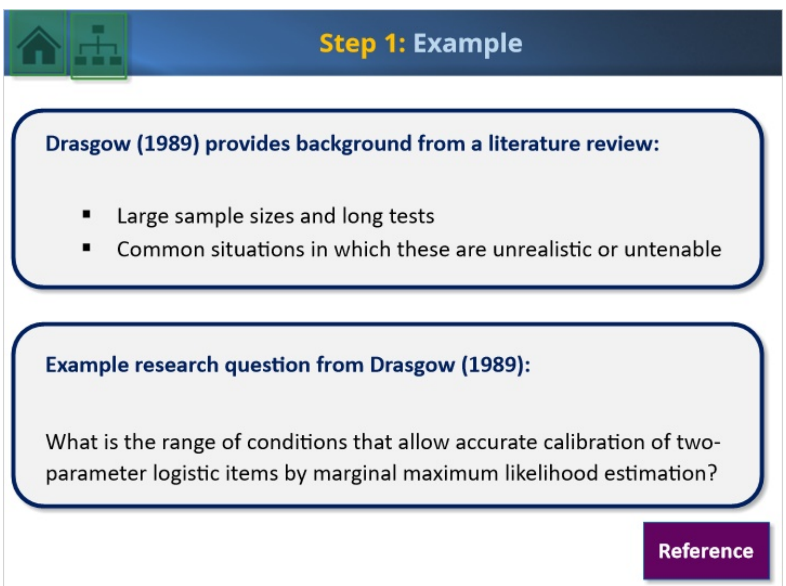
2.15 Step Selection



2.16 Research Questions (I)



2.17 Research Questions (II)



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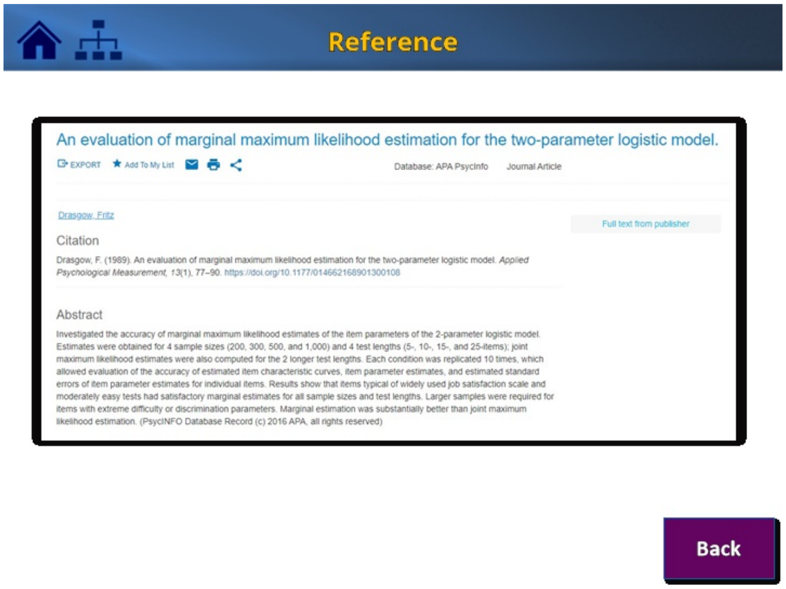
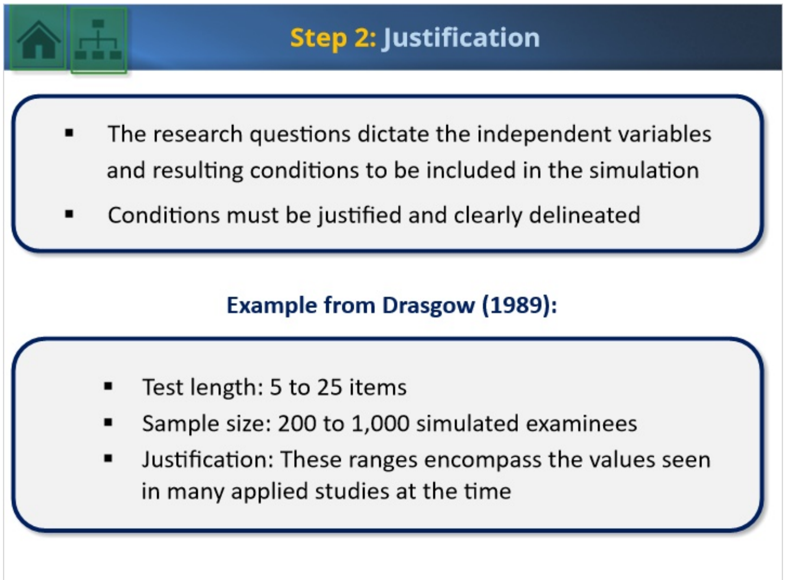


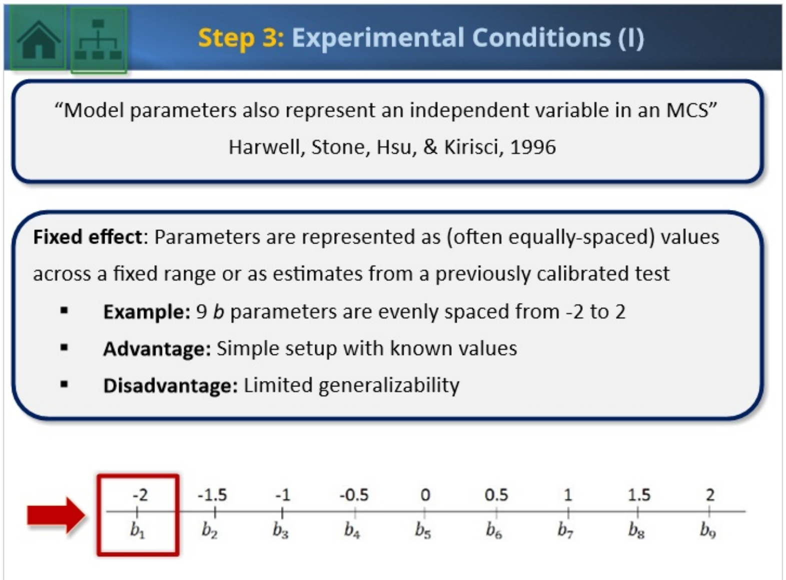
2.18 Bookend: Step 1

2.19 Justifying Conditions

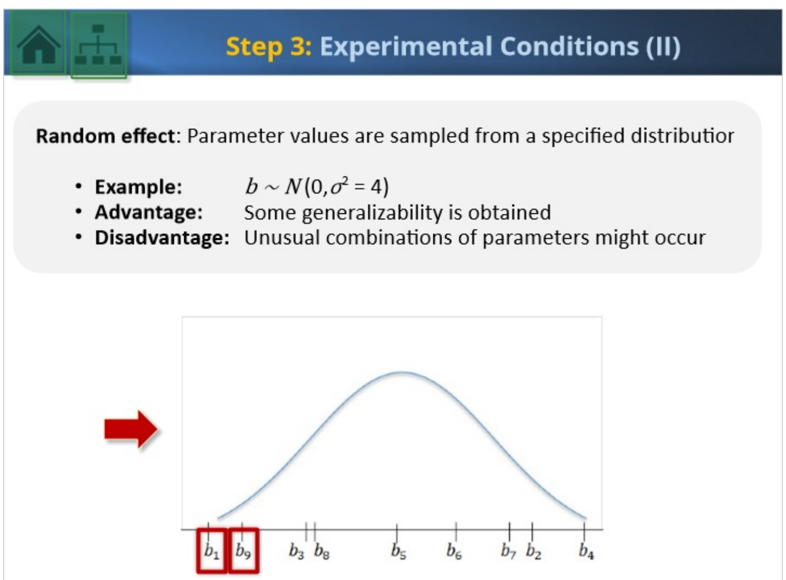


2.20 Bookend: Step 2

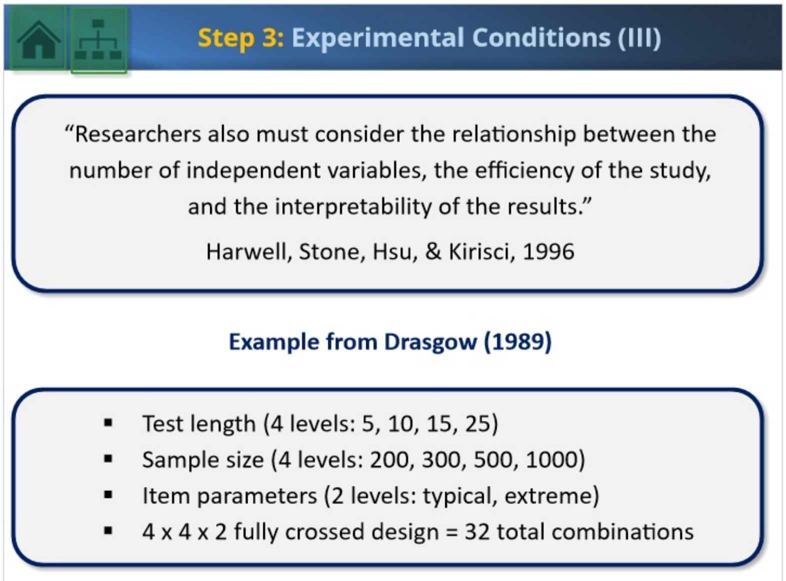
2.21 Experimental Design (I)



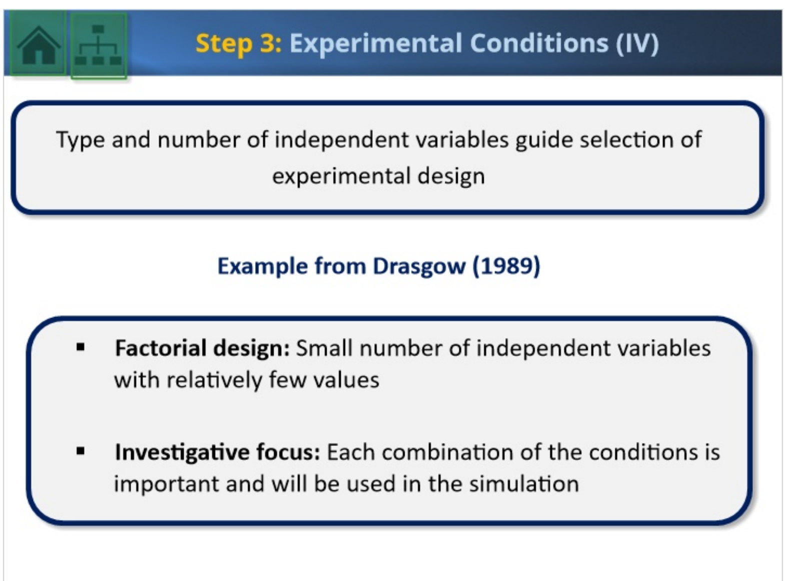
2.22 Experimental Design (II)



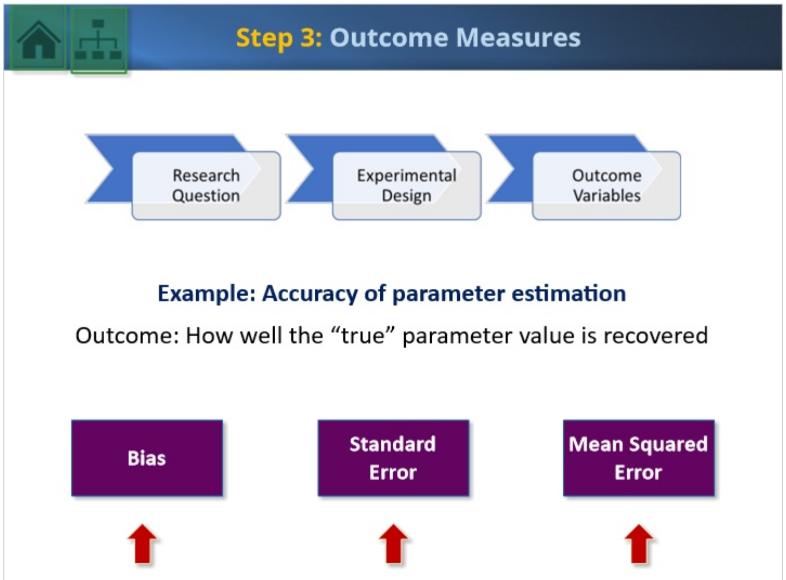
2.23 Experimental Design (III)



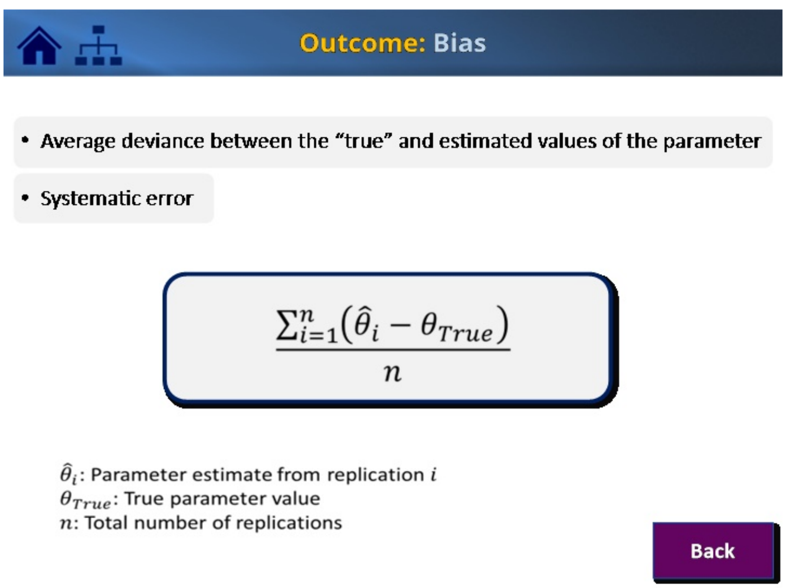
2.24 Experimental Design (IV)



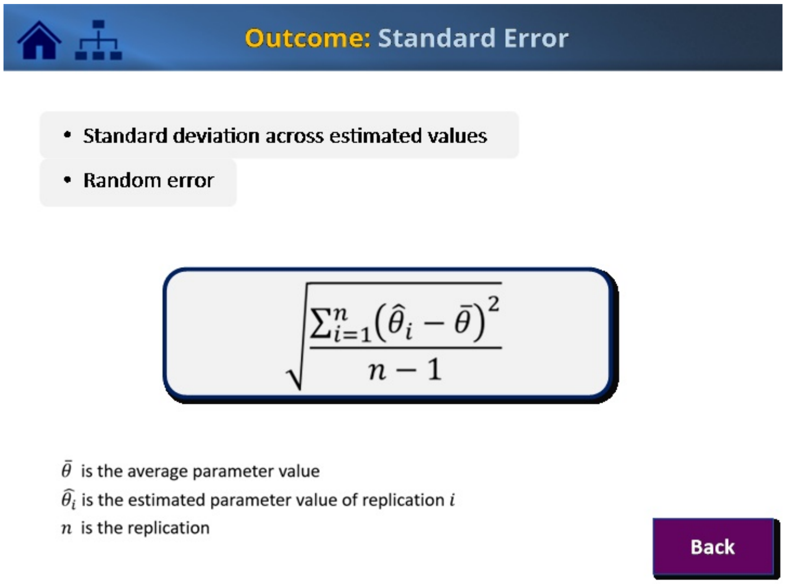
2.25 Experimental Design (V)



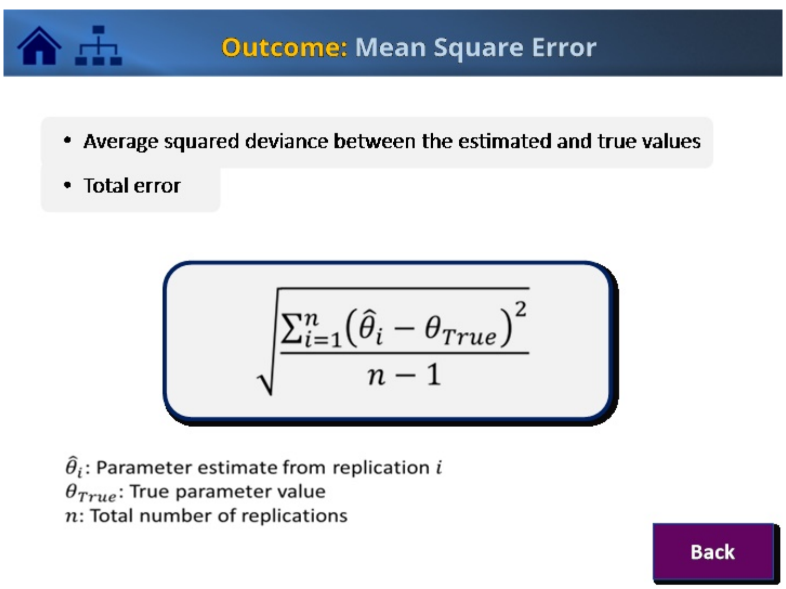
Bias (Slide Layer)



Standard Error (Slide Layer)

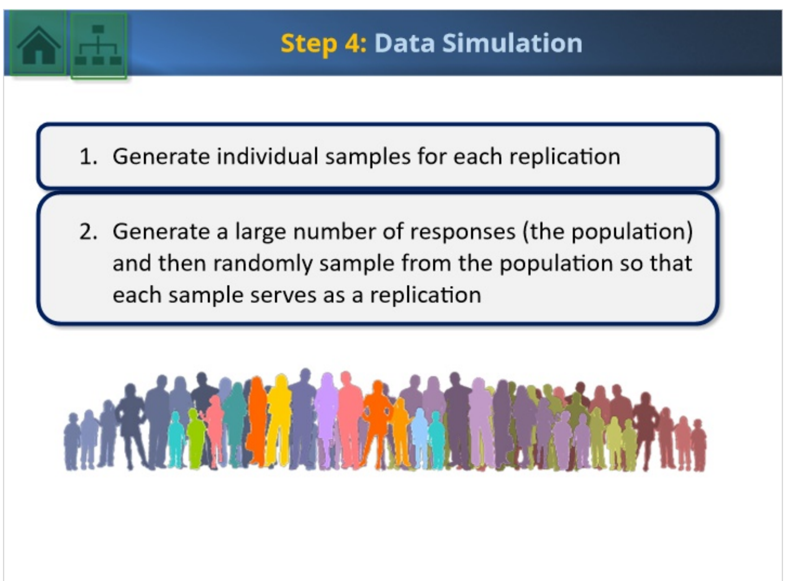


Mean Squared Error (Slide Layer)

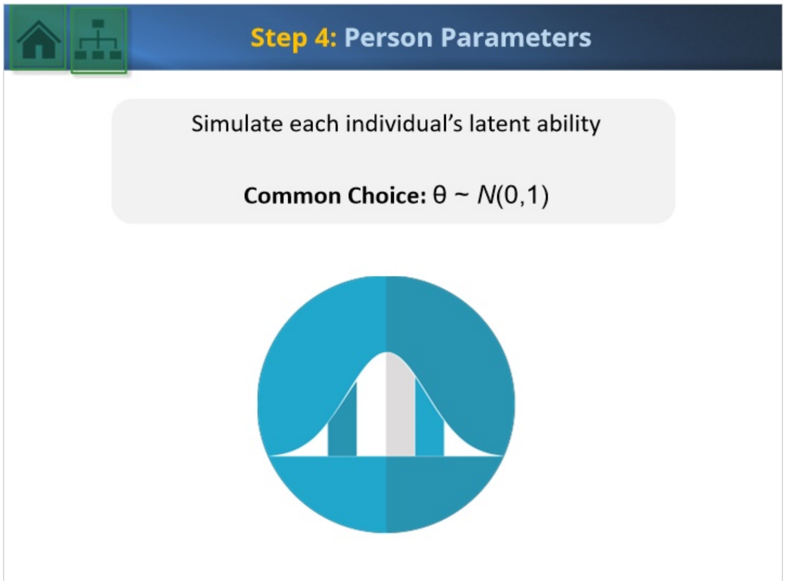


2.26 Bookend: Step 3

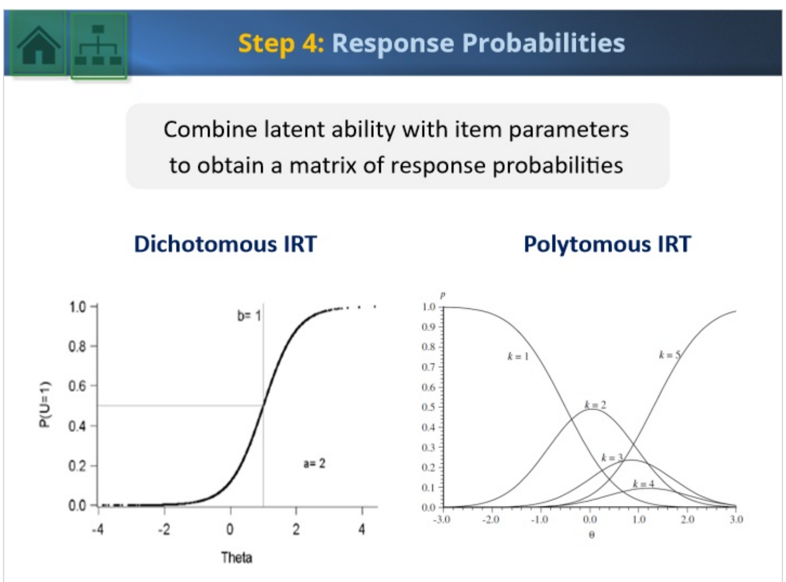
2.27 Simulating Data (I)



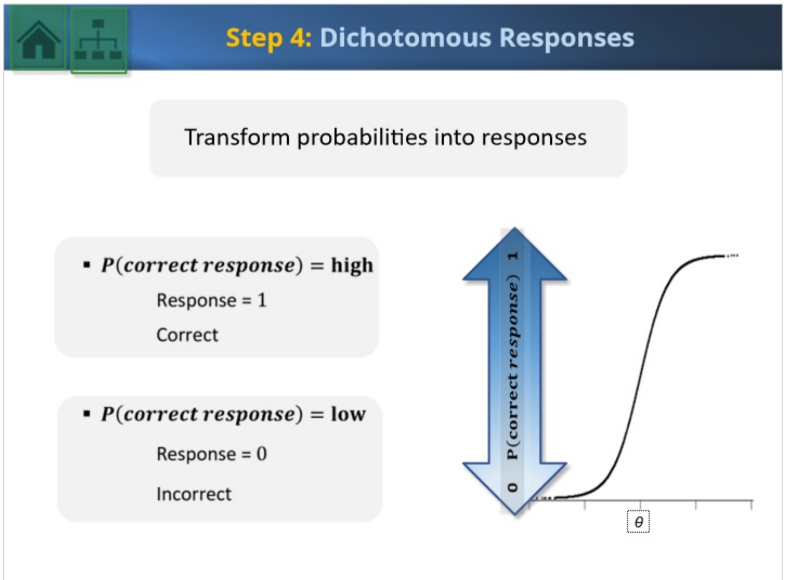
2.28 Simulating Data (II)



2.29 Simulating Data (III)



2.30 Simulating Data (IV)



2.31 Bookend: Step 4

2.32 Estimating Parameters (I)

Step 5: Parameter Estimation

- **Available tools:**
  - ✓ Commercially available programs (e.g., Winsteps or Bilog)
  - ✓ Open source packages (e.g., R package ltm or Stan)
  - ✓ Their own routine (e.g., Fortran, SAS, or Basic)

- **Adequacy** of the estimation algorithm must be documented

- **Validation evidence examples:**
  - ✓ Using the program to analyze a well-known dataset
  - ✓ Analyzing item responses with perfect fit to an item response function in which case the estimation routine should return the exact item parameters

2.33 Estimating Parameters (II)

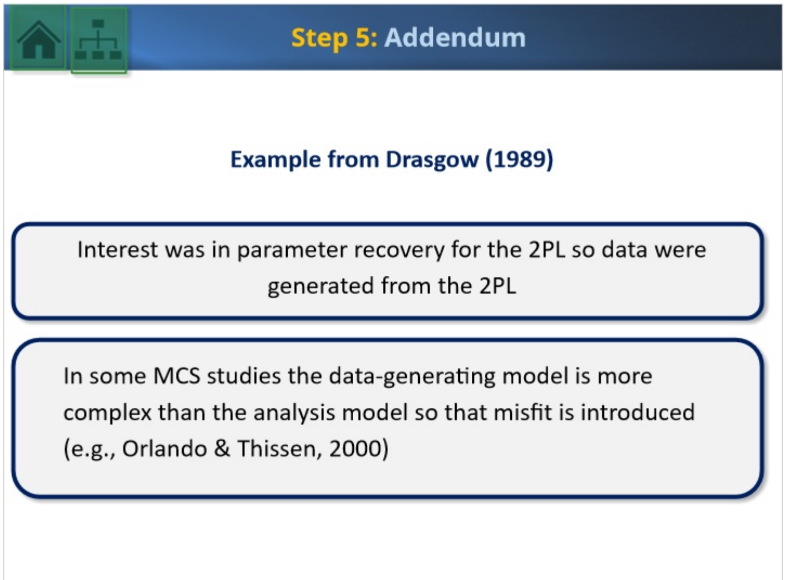
Step 5: Example

Example from Drasgow (1989)

- A FORTRAN program was written to estimate the *a* (discrimination) and *b* (difficulty) parameters of the 2PL

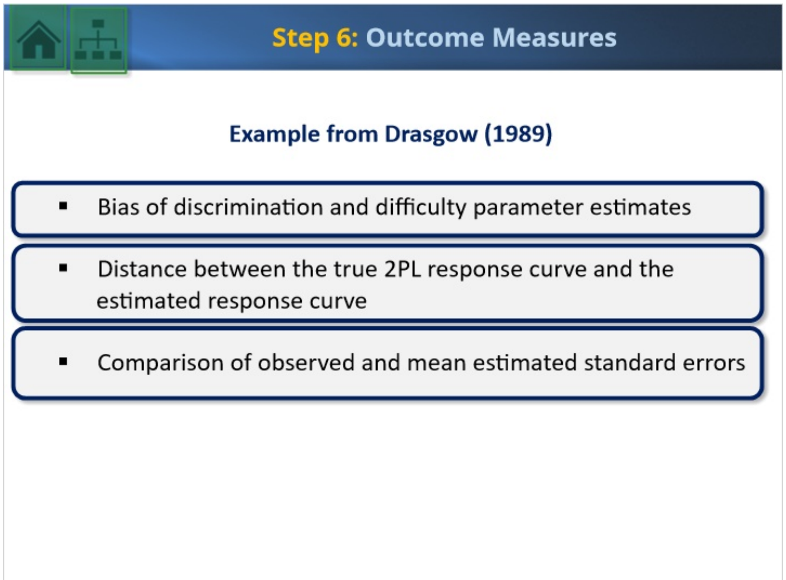
- Validity evidence was not provided

2.34 Estimating Parameters (III)



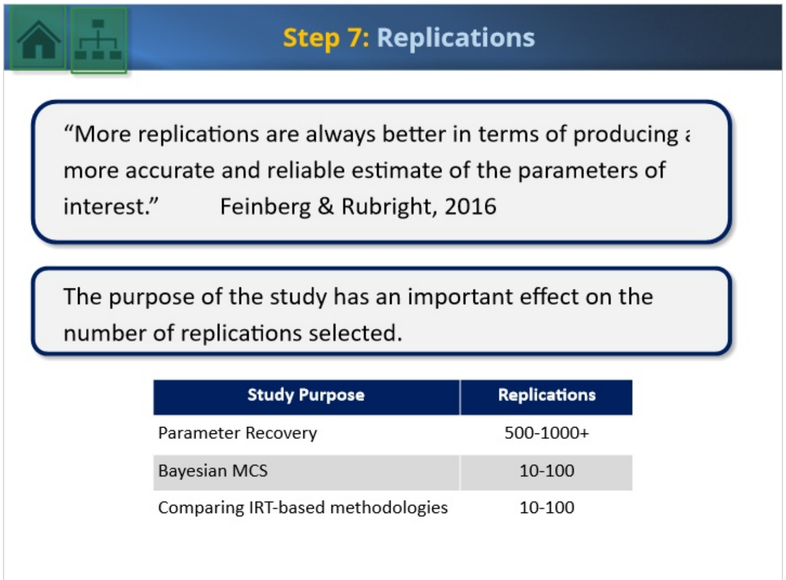
2.35 Bookend: Step 5

2.36 Outcome Measures



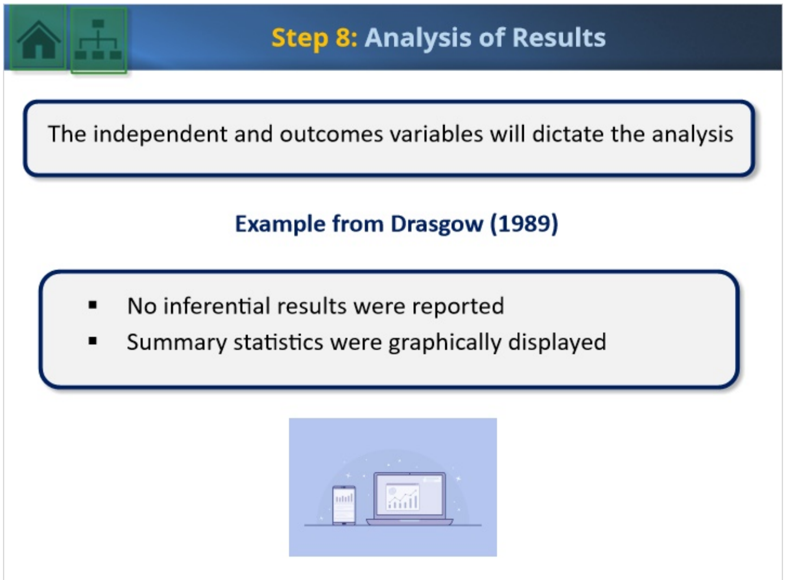
2.37 Bookend: Step 6

2.38 Replications



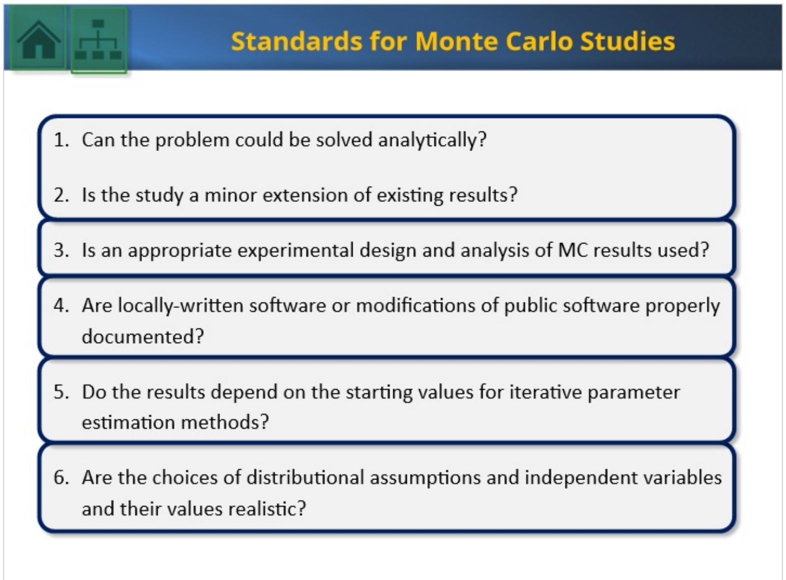
2.39 Bookend: Step 7

2.40 Analyzing Results

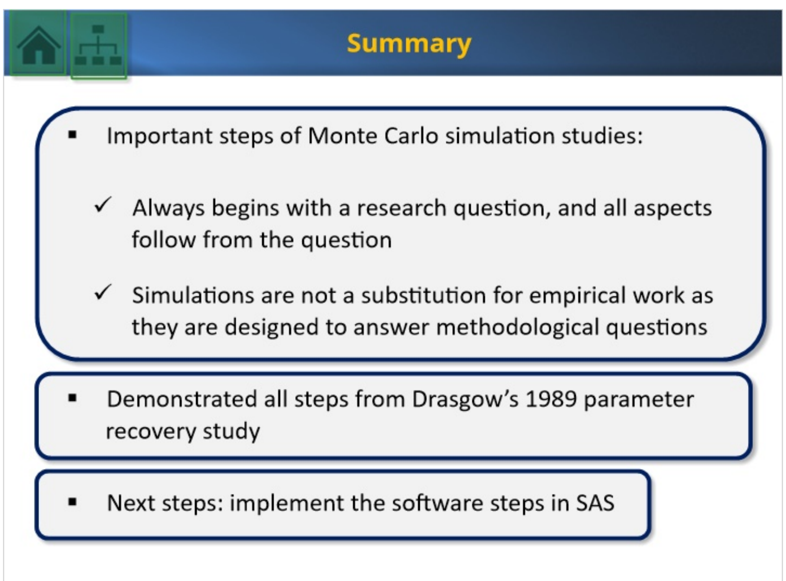


2.41 Bookend: Step 8

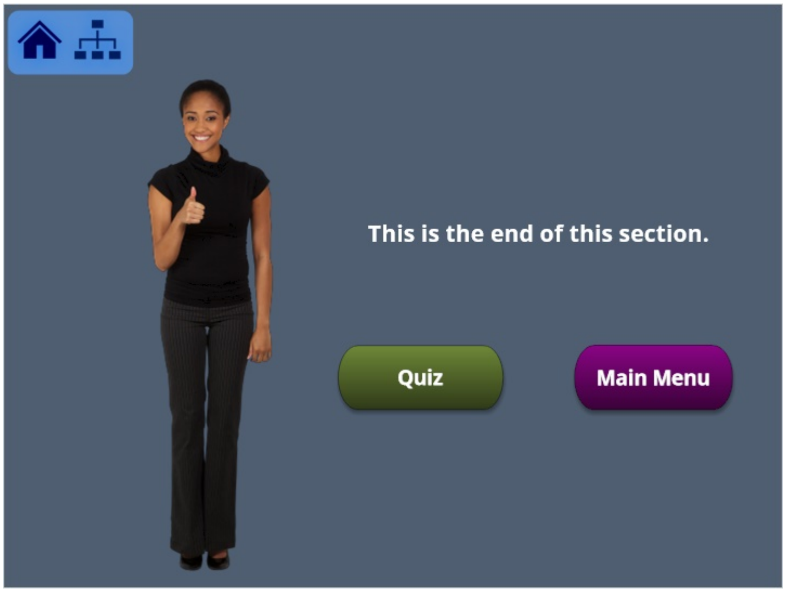
2.42 Simulation Standards



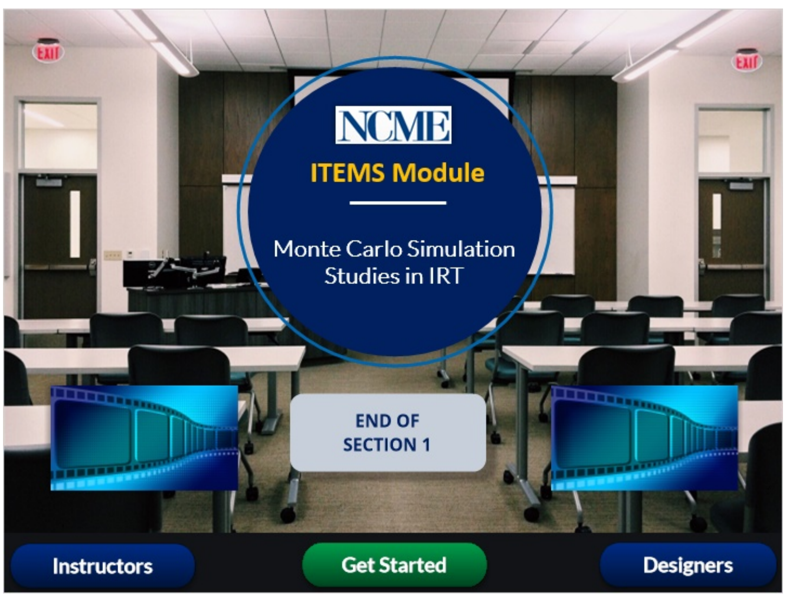
2.43 Summary



2.44 Bookend: Section 1

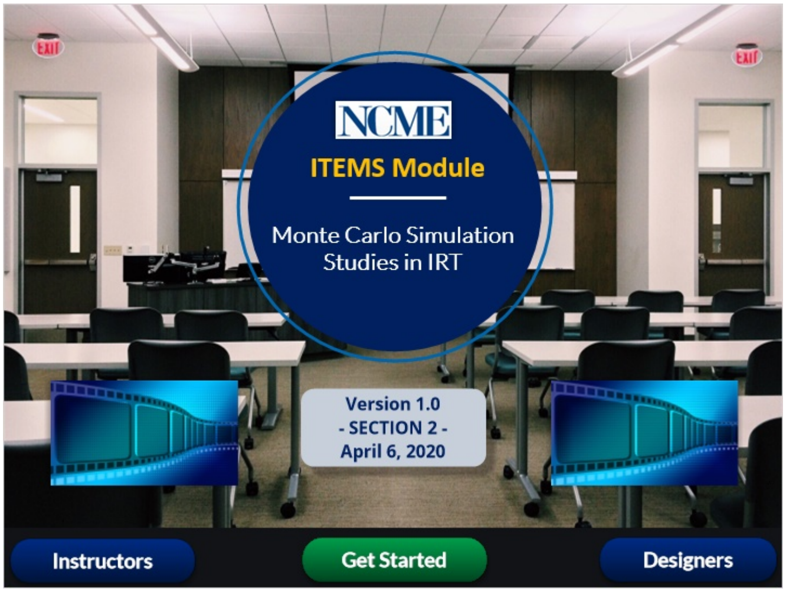


2.45 Module Cover (END)

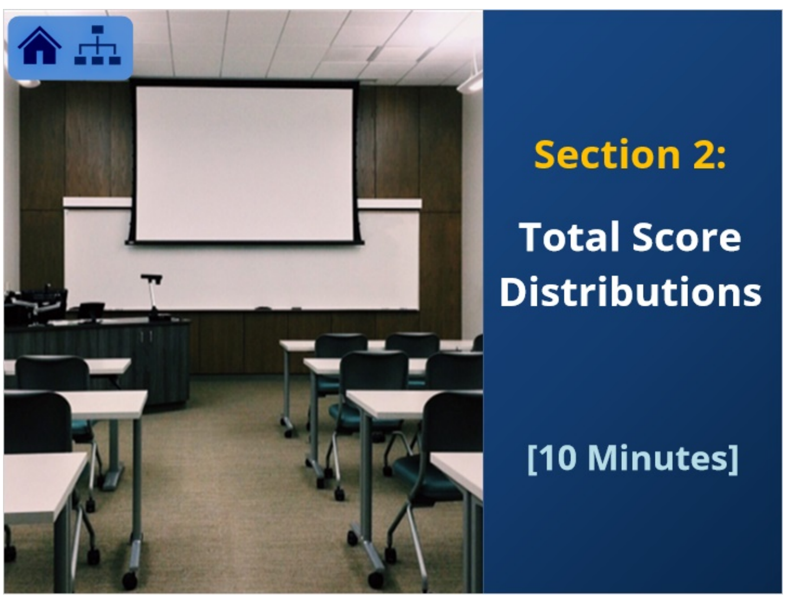


3. Section 2: Total Score Distributions

3.1 Module Cover (START)

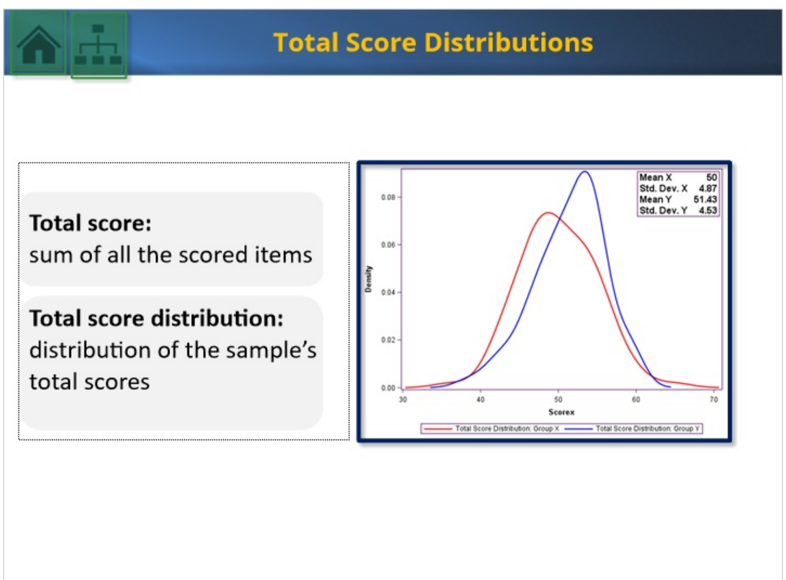


3.2 Cover: Section 2

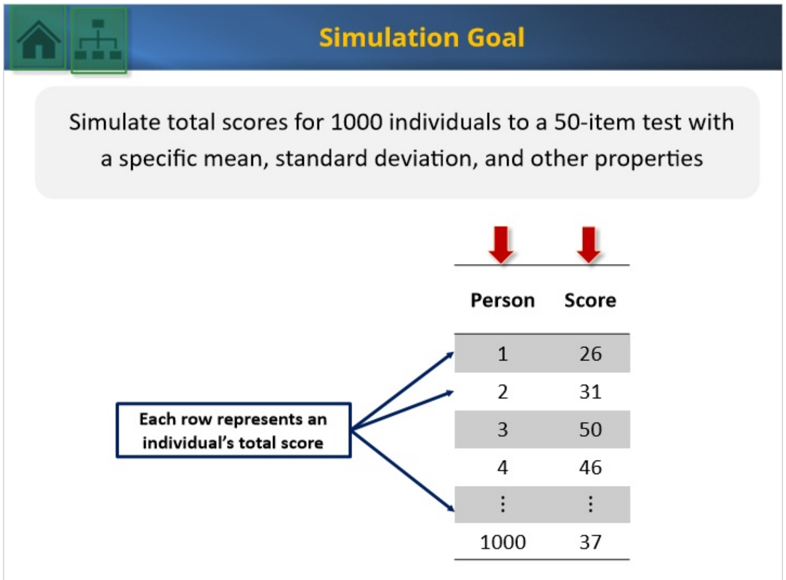


3.3 Objectives

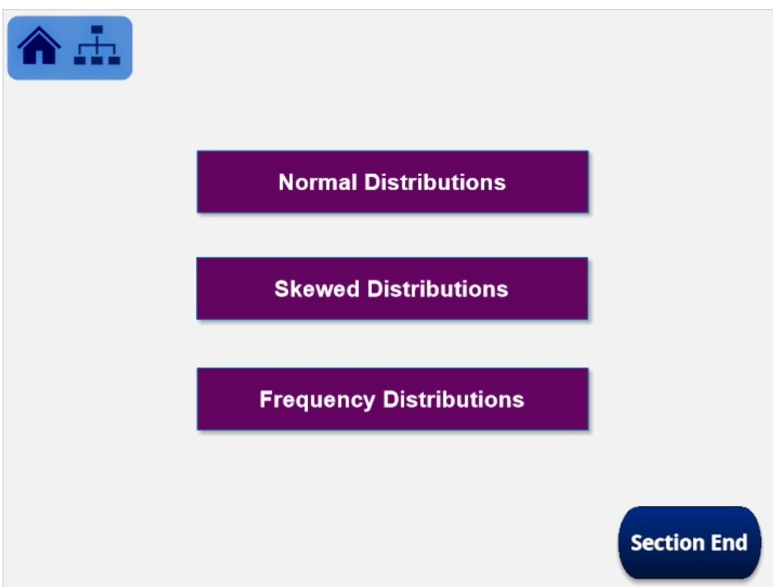
3.4 Total Score Distributions



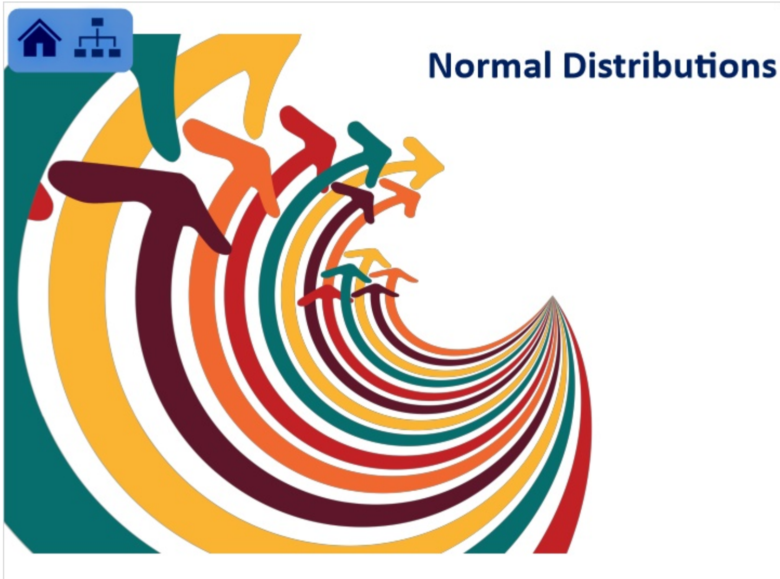
3.5 Simulation Goal



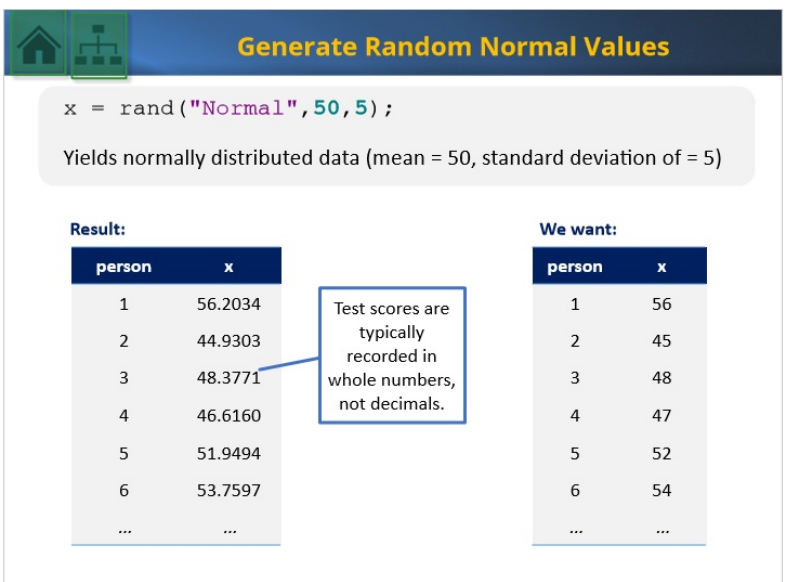
3.6 Topic Selection



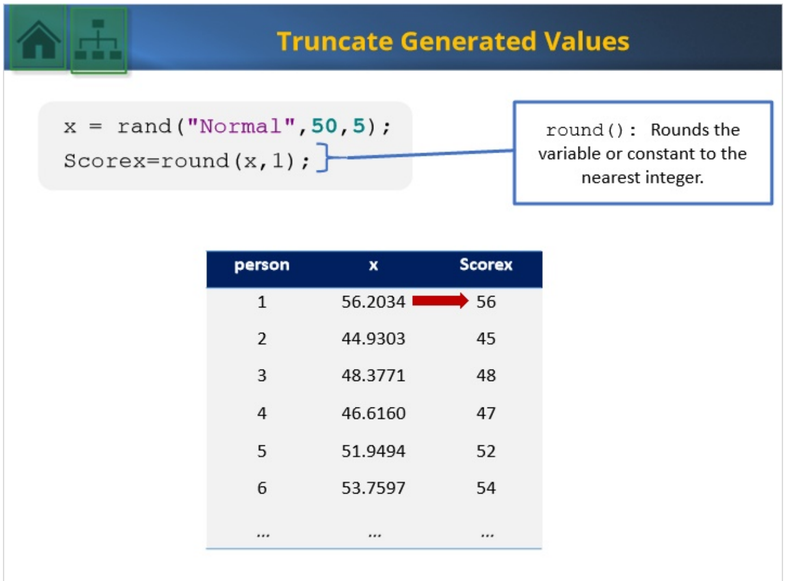
3.7 Bookmark: Normally Distributed Data



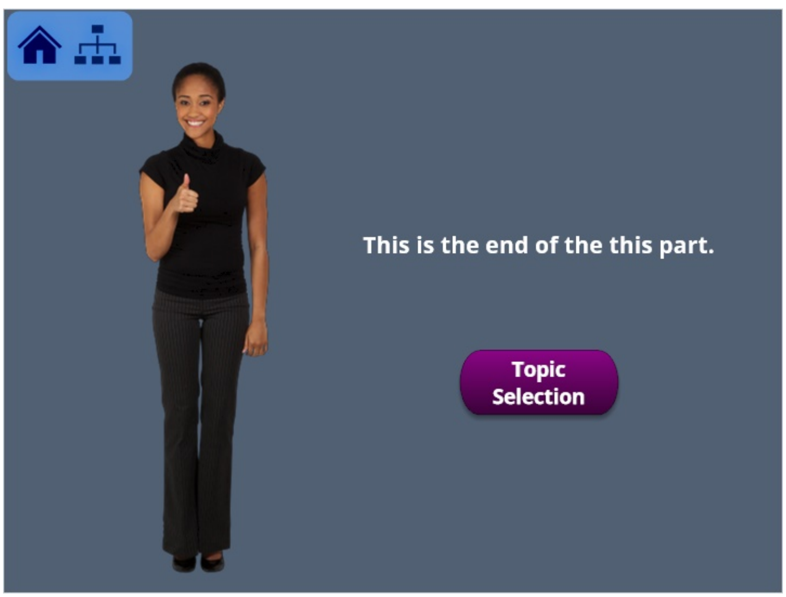
3.8 Standard Normal (I)



3.9 Standard Normal (II)

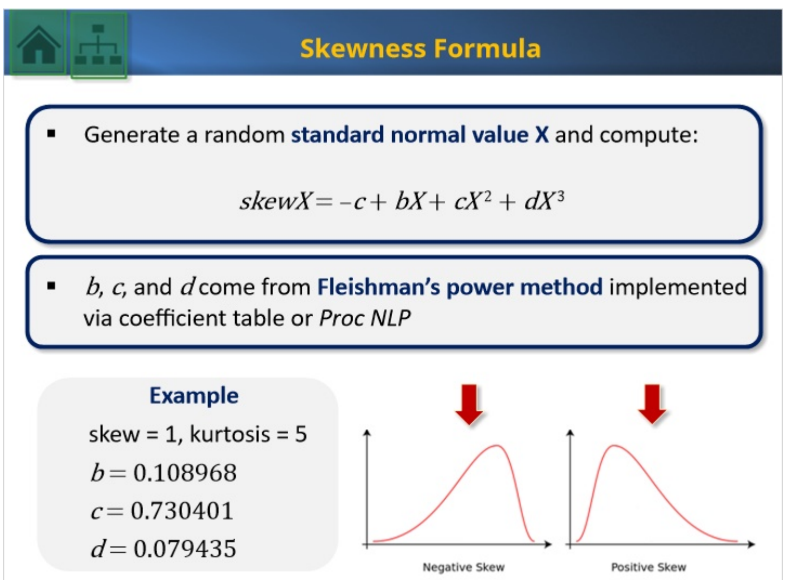


3.10 Bookend: Normal Distributions

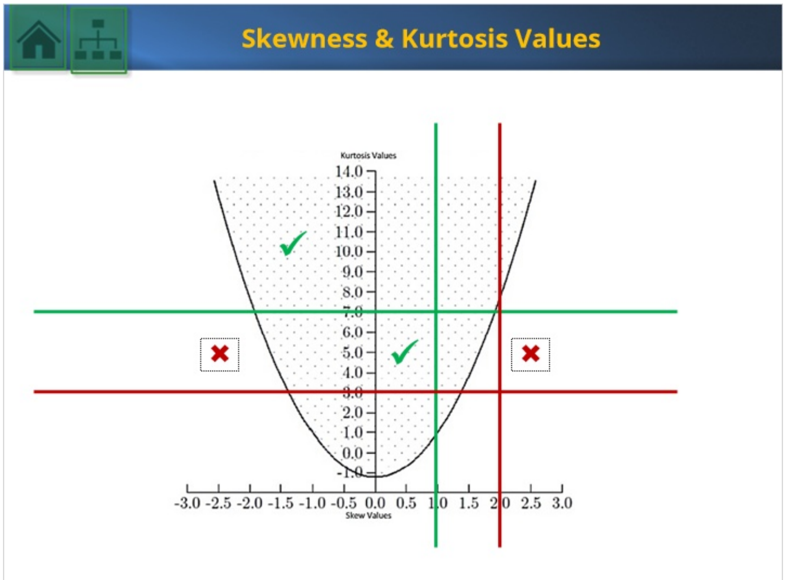


3.11 Bookmark: Skewed Data

3.12 Skewed Distribution (I)



3.13 Skewed Distribution (II)



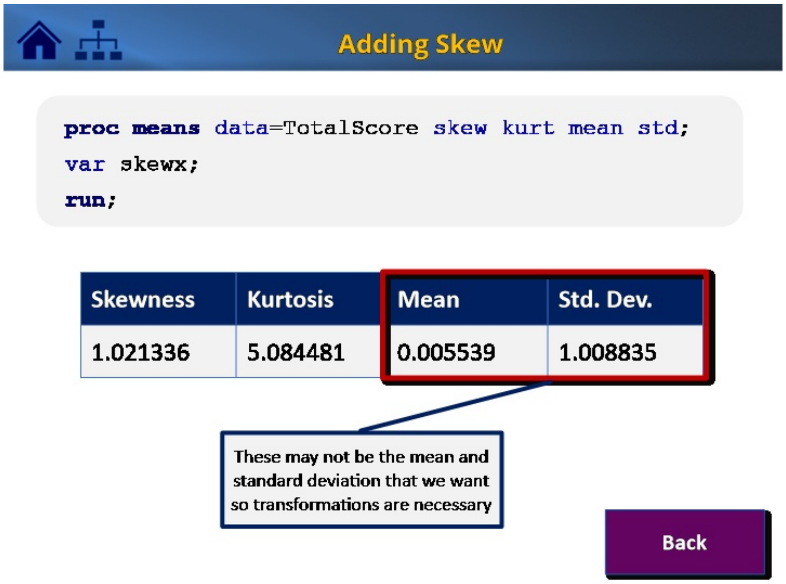
3.14 Skewed Distribution (III)

Skewed Distribution

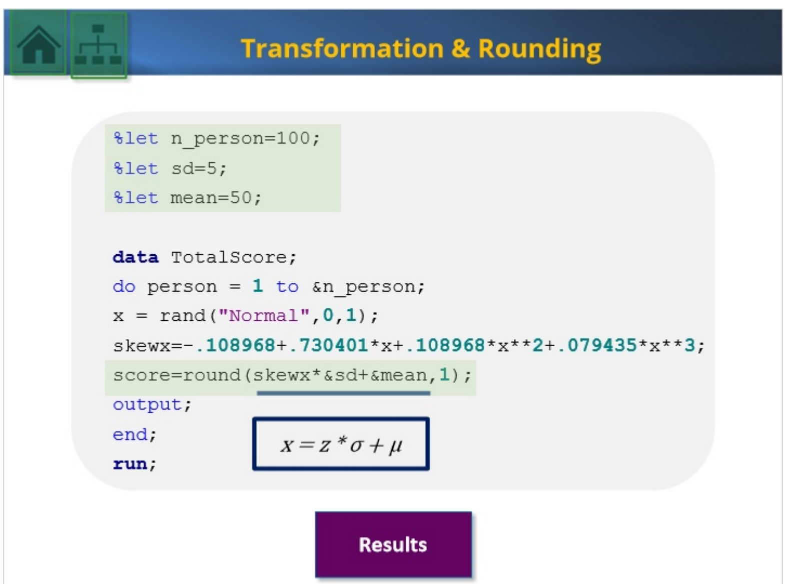
```
%let n_person=100;

data TotalScore;
do person = 1 to &n_person;
x = rand("Normal",0,1);
skewx=-.108968+.730401*x+.108968*x**2+.079435*x**3;
output;
end;
run;
```

Results

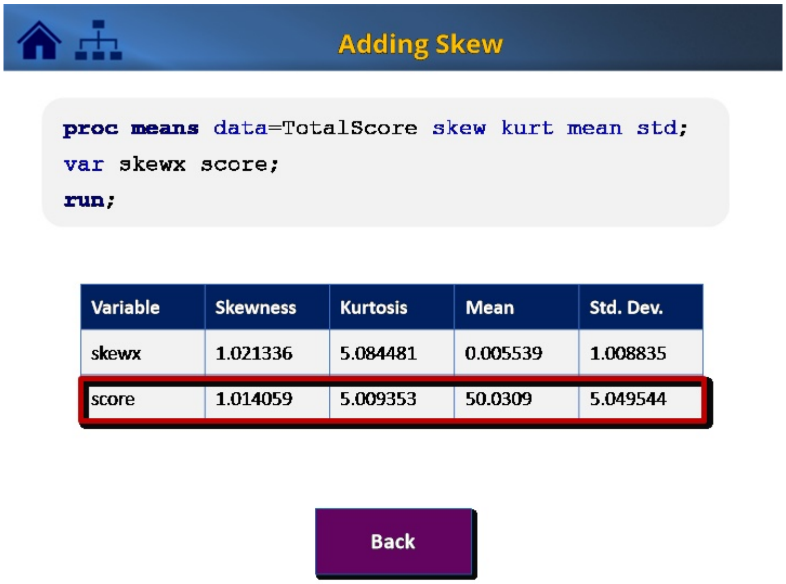
Results (Slide Layer)



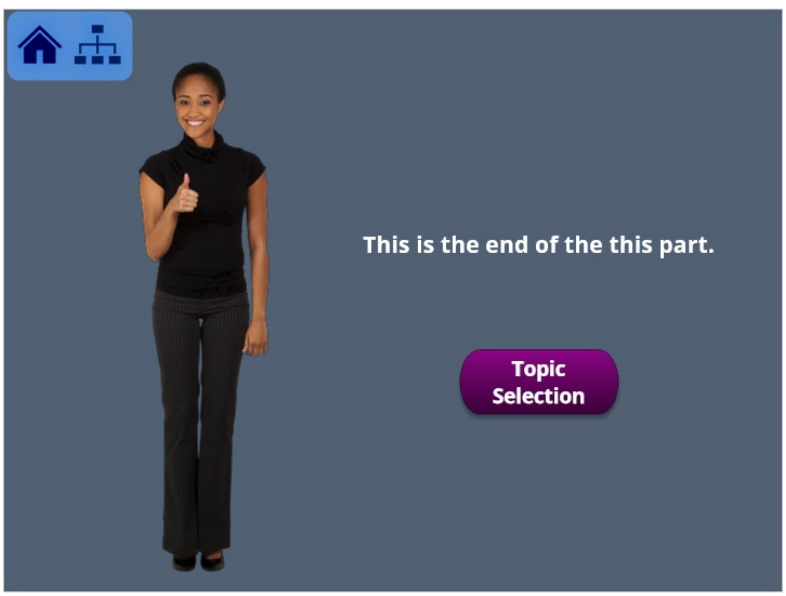
3.15 Skewed Distribution (IV)



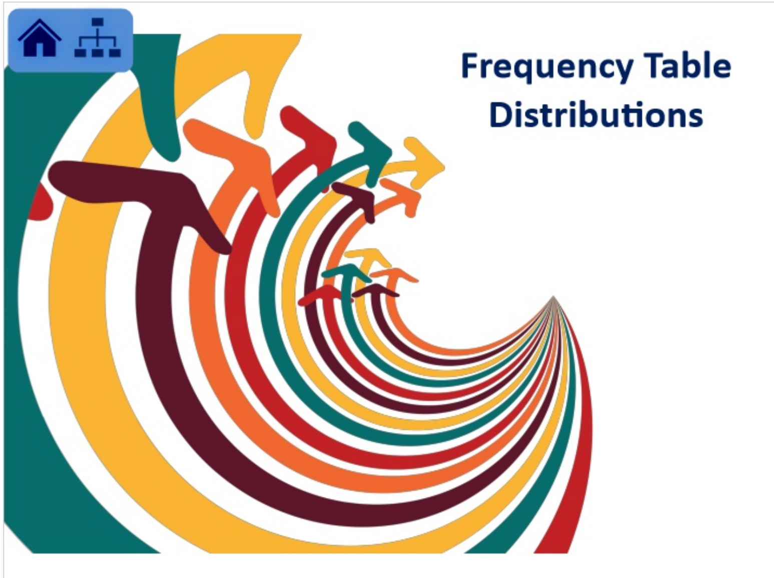
Results (Slide Layer)



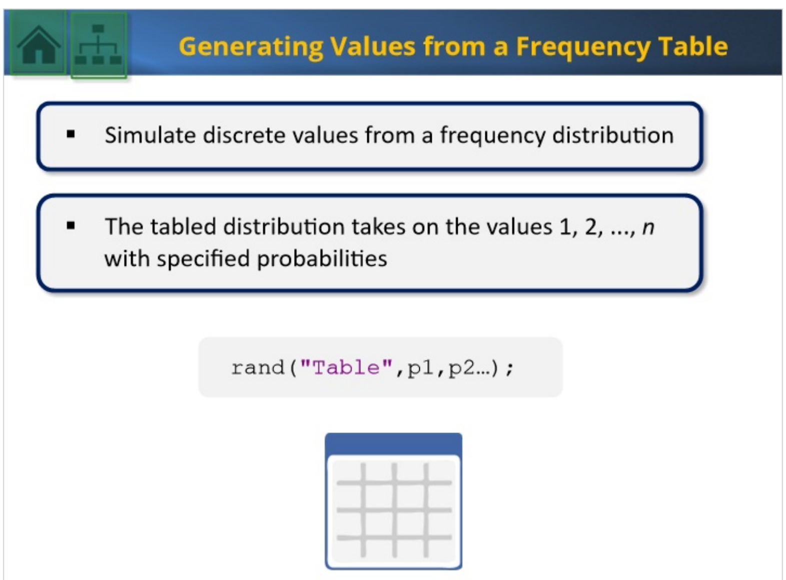
3.16 Bookend: Skewed Distributions



3.17 Bookmark: Data from Freq Distribution



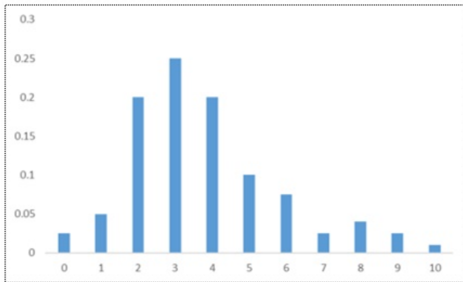
3.18 Frequency Table (I)



3.19 Frequency Table (II)

Example

Score	p
0	0.025
1	0.050
2	0.200
3	0.250
4	0.200
5	0.100
6	0.075
7	0.025
8	0.040
9	0.025
10	0.010



3.20 Frequency Table (III)

Implementation

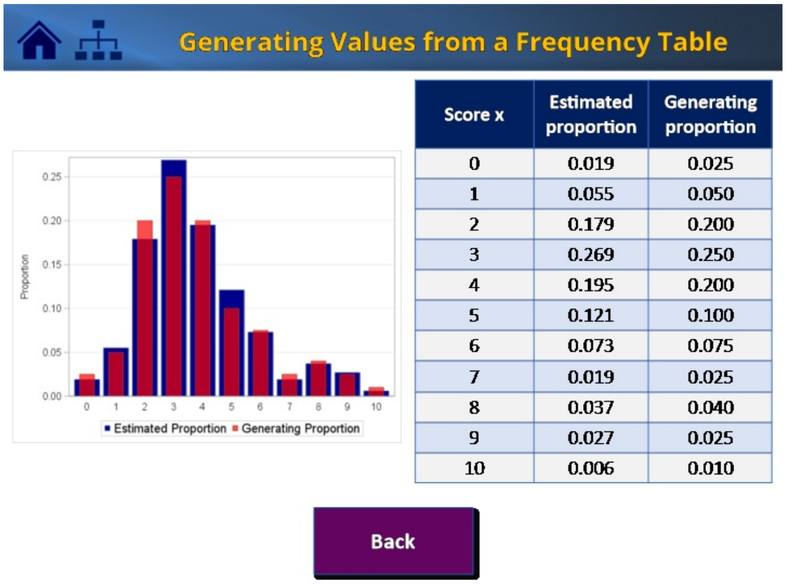
```
data CatScore;
do person = 1 to &n_person;
Score = rand("Table",
0.025,0.05,0.2,0.25,0.2,0.1,0.075,0.025,0.04,0.025,0.01);

Scorex=Score-1;

output;
end;
run;
```

Vector of the probability of scores from 0 to 1

SAS simulates scores from 1 to 11 but we want them from 0 to 10 so we need to subtract 1

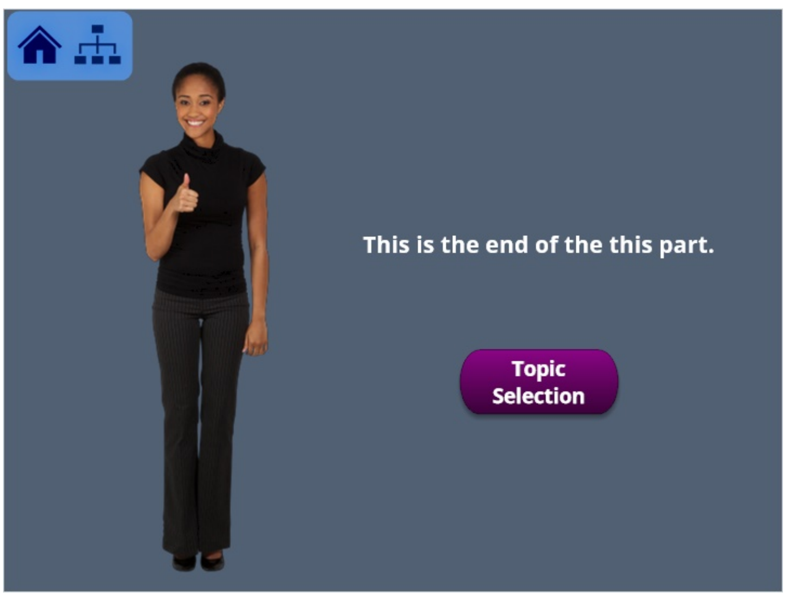
Results

Results (Slide Layer)

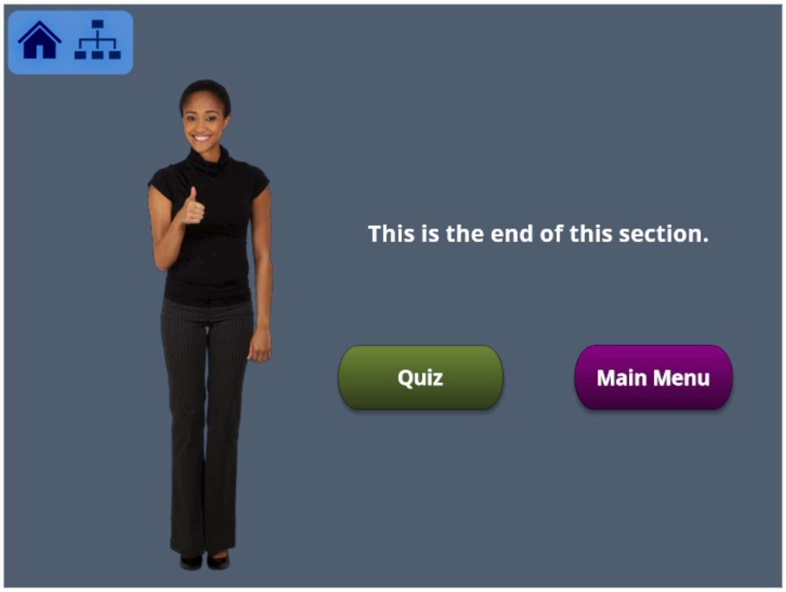


Score x	Estimated proportion	Generating proportion
0	0.019	0.025
1	0.055	0.050
2	0.179	0.200
3	0.269	0.250
4	0.195	0.200
5	0.121	0.100
6	0.073	0.075
7	0.019	0.025
8	0.037	0.040
9	0.027	0.025
10	0.006	0.010

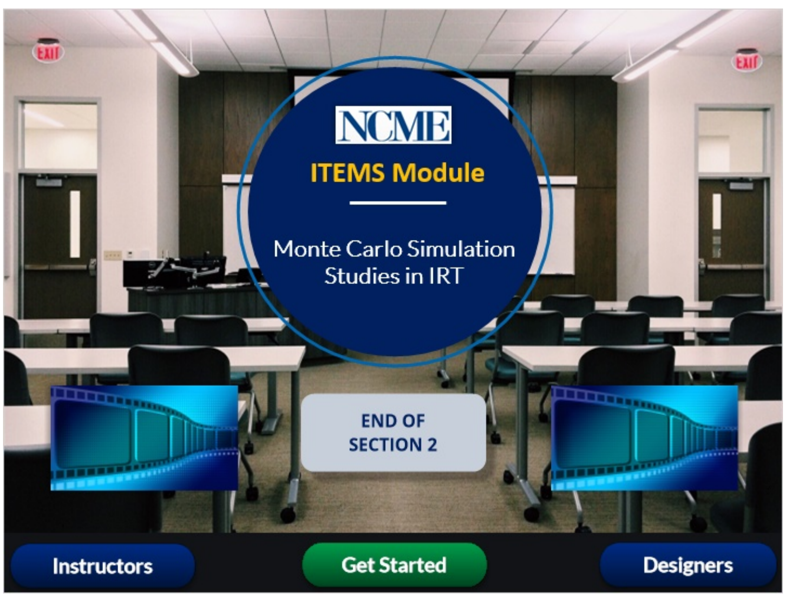
3.21 Bookend: Frequency Table Distributions



3.22 Bookend: Section 2

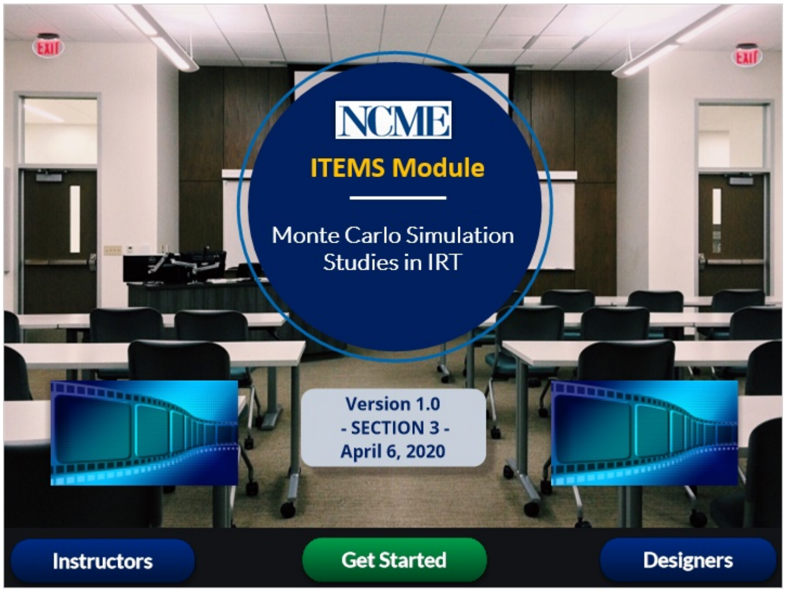


3.23 Module Cover (END)

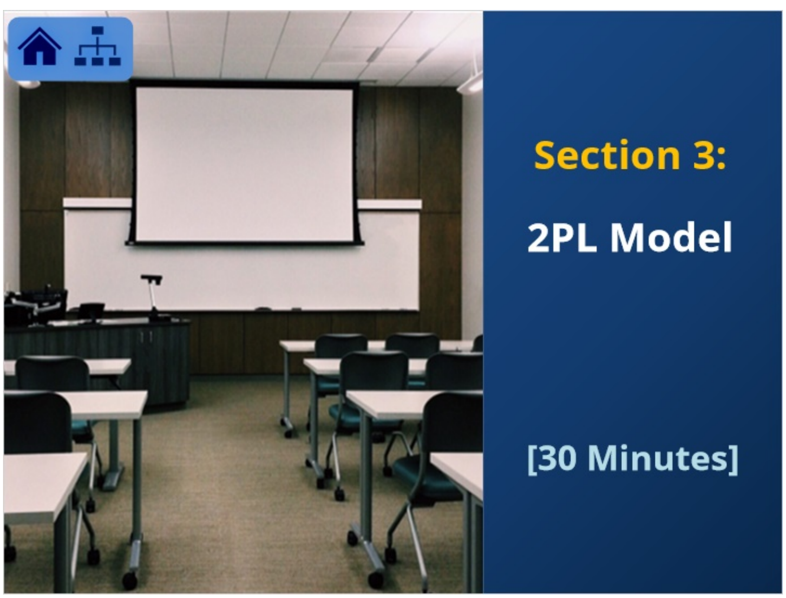


4. Section 3: Two-parameter Model

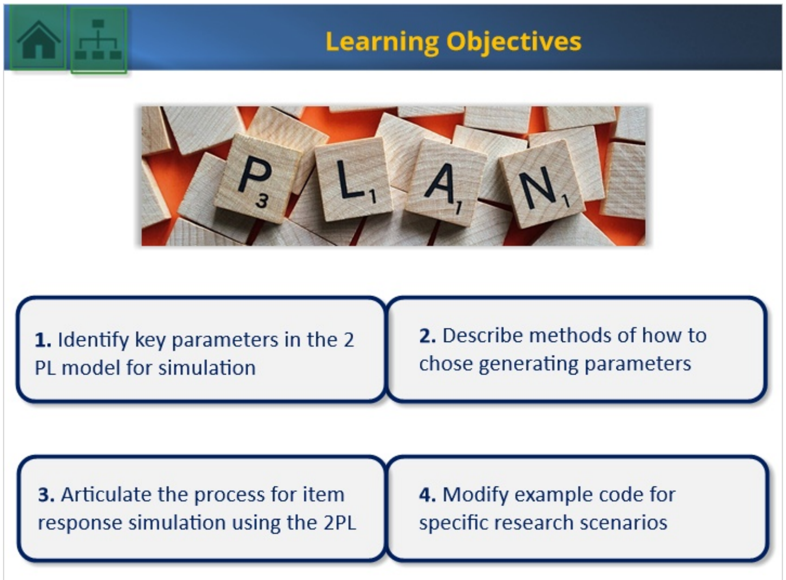
4.1 Module Cover (START)



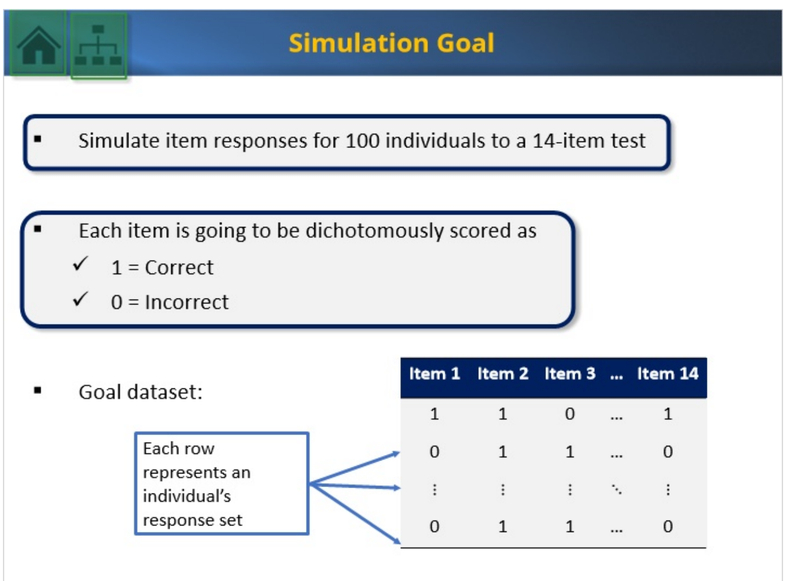
4.2 Cover: Section 3



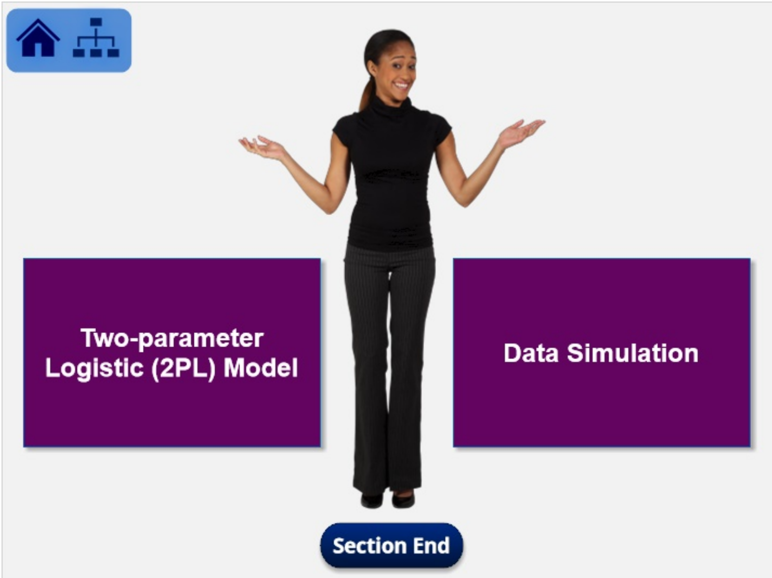
4.3 Objectives



4.4 Simulation Goal



4.5 Topic Selection



4.6 Bookmark: Two-paramter Model

4.7 Two-parameter Model (I)

Two-Parameter Logistic (2PL) Model (I)

$$P_j(U_j = 1|\boldsymbol{\omega}_j, \theta) = \frac{1}{1 + e^{-Da_j(\theta - b_j)}}$$

- Probability an individual with a given ability θ correctly answers item j that has item parameters $\boldsymbol{\omega}_j$
- U_j: response on item j (1 = correct, 0 = incorrect)
- $\boldsymbol{\omega}_j$ is the vector of all item parameters
- θ is an ability value on the trait of interest

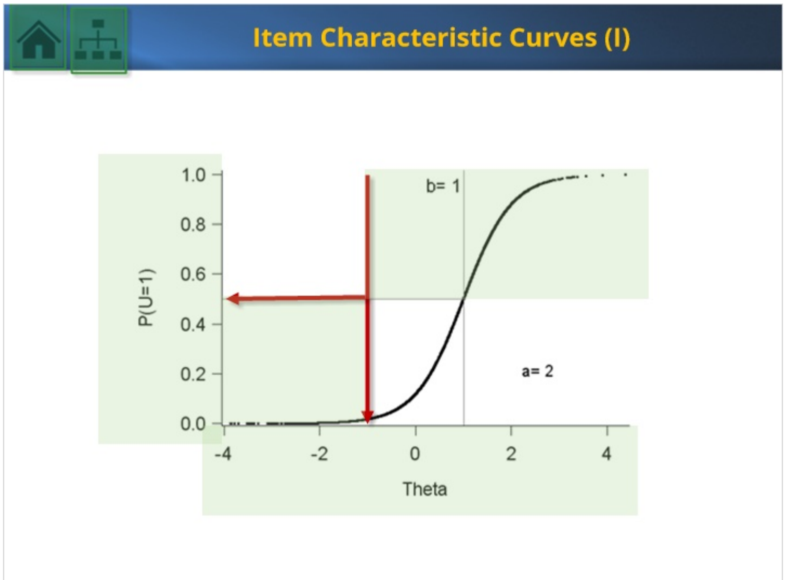
4.8 Two-parameter Model (II)

Two-Parameter Logistic (2PL) Model (II)

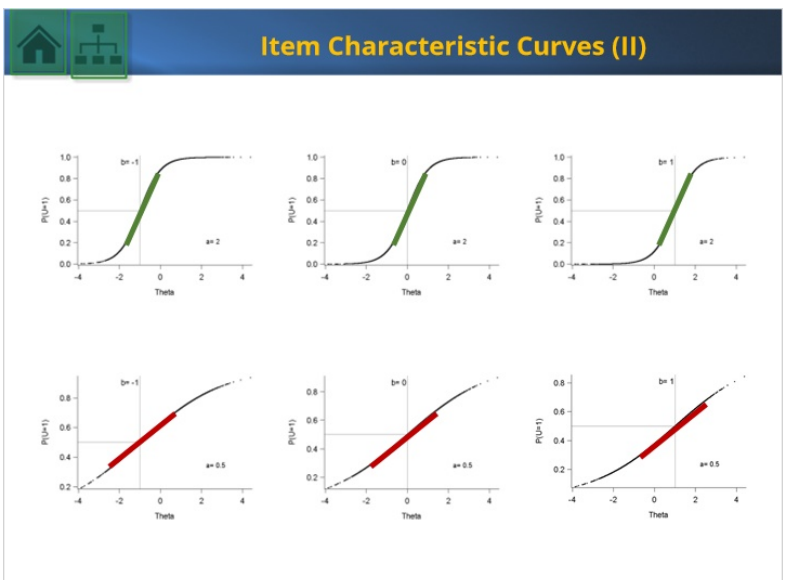
$$P_j(U_j = 1|\boldsymbol{\omega}_j, \theta) = \frac{1}{1 + e^{-Da_j(\theta - b_j)}}$$

- e is Euler's number (approx. 2.718)
- D is the scaling parameter
  - ✓ Here we set $D = 1$
  - ✓ Other common value is $D = 1.702$
- a_j is the discrimination parameter for item j
- b_j is the difficulty/threshold parameter for item j

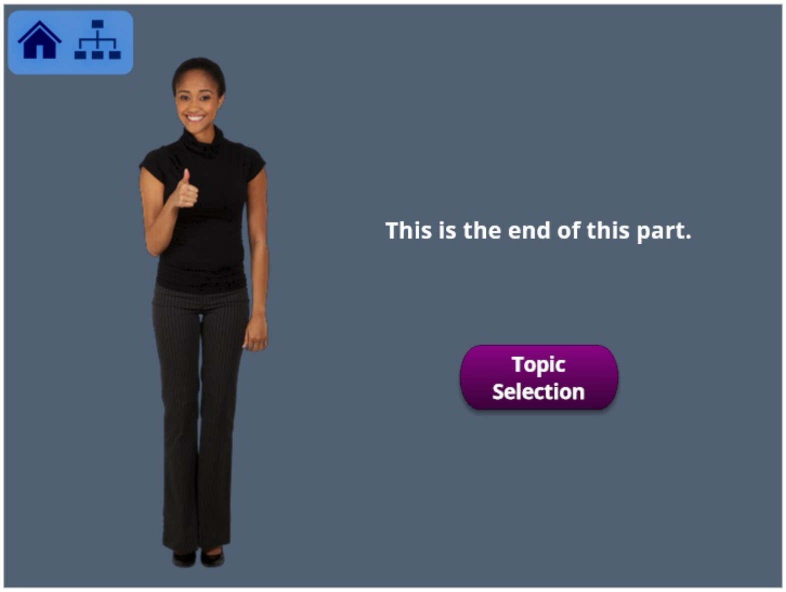
4.9 Item Characteristic Curves (I)



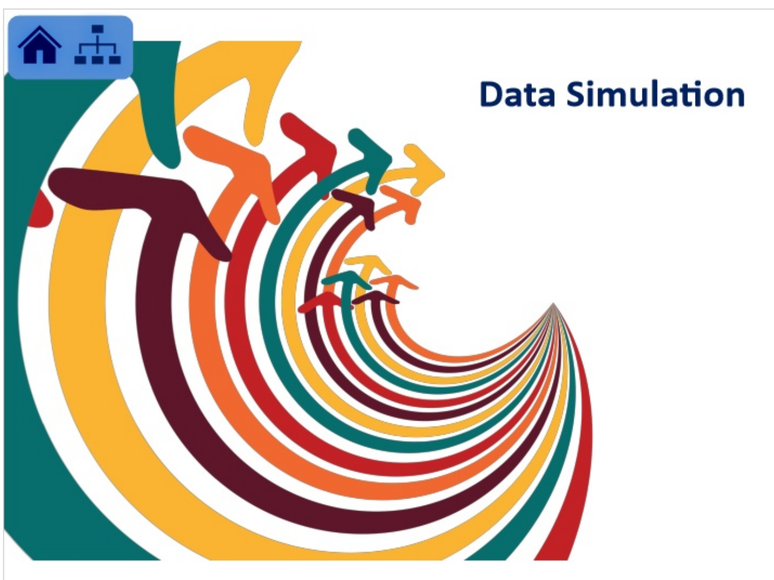
4.10 Item Characteristic Curves (II)



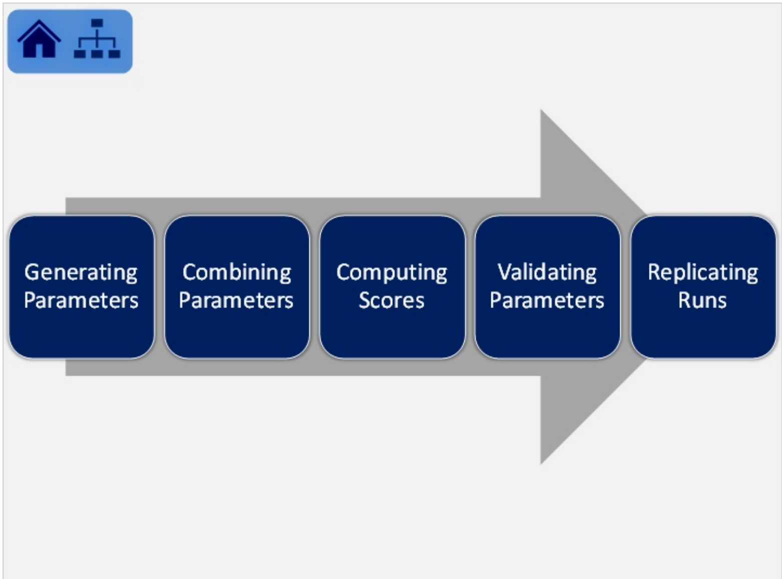
4.11 Bookend: Two-parameter Model



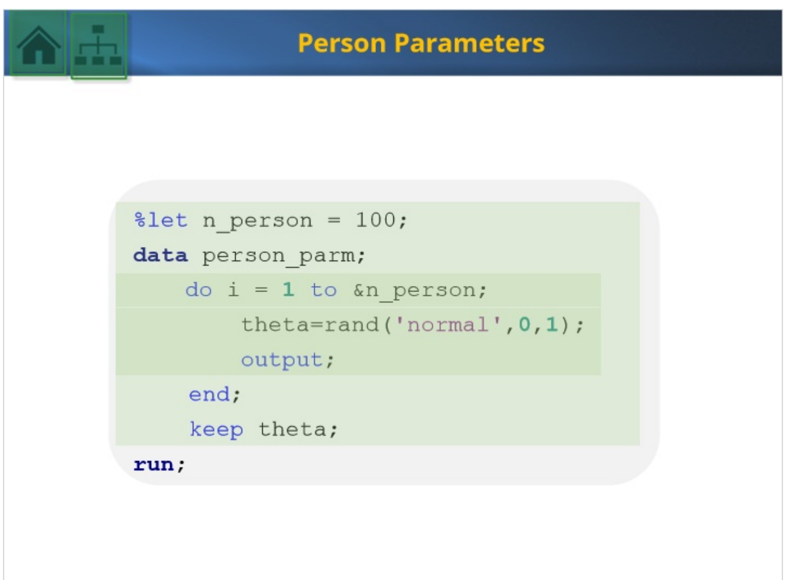
4.12 Bookmark: Generating Parameters with 2PLM



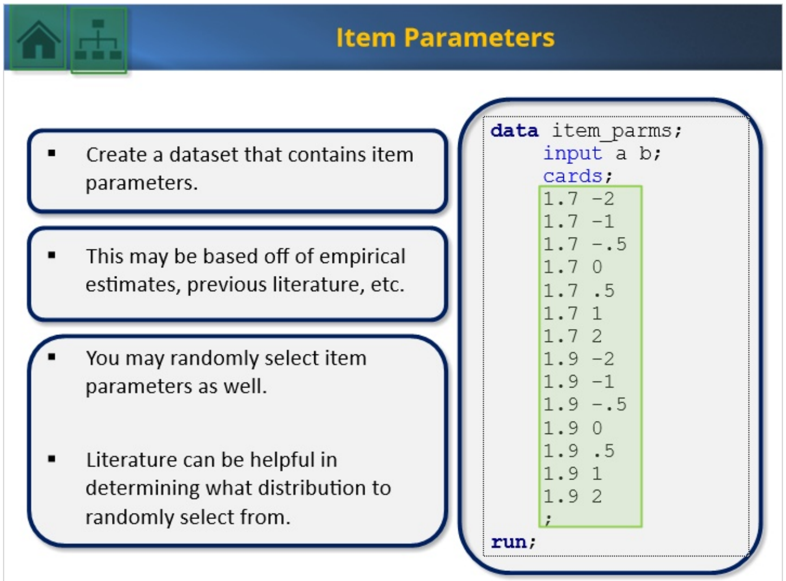
4.13 Step Selection



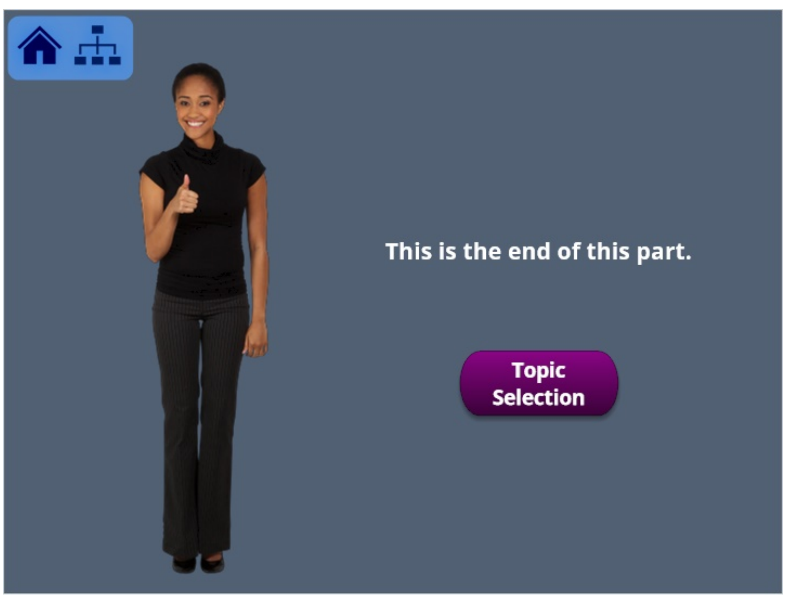
4.14 Person Parameters



4.15 Item Parameters



4.16 Bookend: Step 1



4.17 Parameter Matrix (I)

Parameter Matrix (I)

- We need in the same data set / parameter matrix:
  - ✓ an individual's ability (θ)
  - ✓ item parameters (a_j and b_j)

- We want our data set / parameter matrix to look like this:

Theta	a_1	b_1	...	a_{14}	b_{14}
-1.43	1.7	-2	...	1.9	2
2.81	1.7	-2	...	1.9	2
⋮	⋮	⋮	⋱	⋮	⋮
0.09	1.7	-2	...	1.9	2

4.18 Parameter Matrix (II)

Parameter Matrix (II)

	a	b
1	1.7	-2
2	1.7	-1
3	1.7	-0.5
4	1.7	0
5	1.7	0.5
6	1.7	1
7	1.7	2
8	1.9	-2
9	1.9	-1
10	1.9	-0.5
11	1.9	0
12	1.9	0.5
13	1.9	1
14	1.9	2

```
proc transpose data=item_parms out=a_wide prefi
      var a;
run;
```

	NAME OF FORMER VARIABLE	a1	a2	a3	a4	a5	a6	a7
1	a	1.7	1.7	1.7	1.7	1.7	1.7	1.7

```
proc transpose data=item_parms out=b_wide prefi
      var b;
run;
```

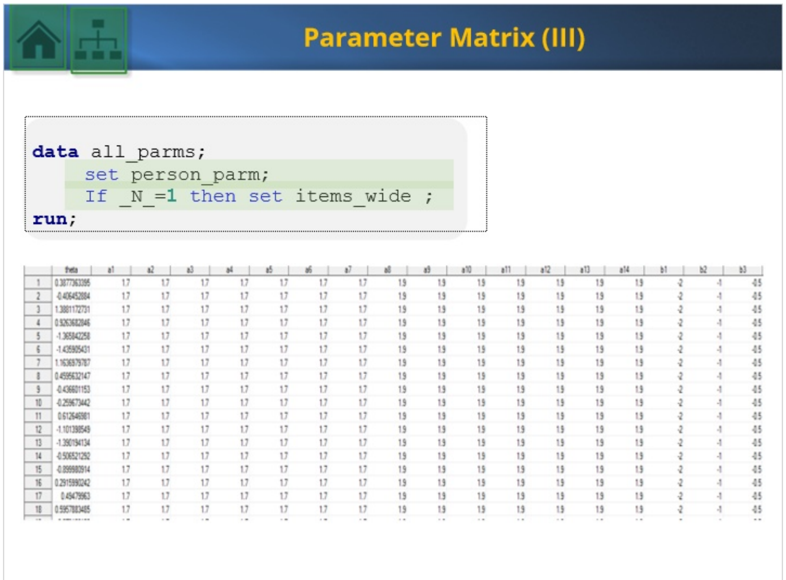
	NAME OF FORMER VARIABLE	b1	b2	b3	b4	b5	b6	b7	b8
1	b	-2	-1	-0.5	0	0.5	1	2	-2

```
data items_wide;
      merge a_wide b_wide;
      drop _name_;
run;
```

Goal:

a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11	a12	a13	a14	b1	b2	b3
1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5

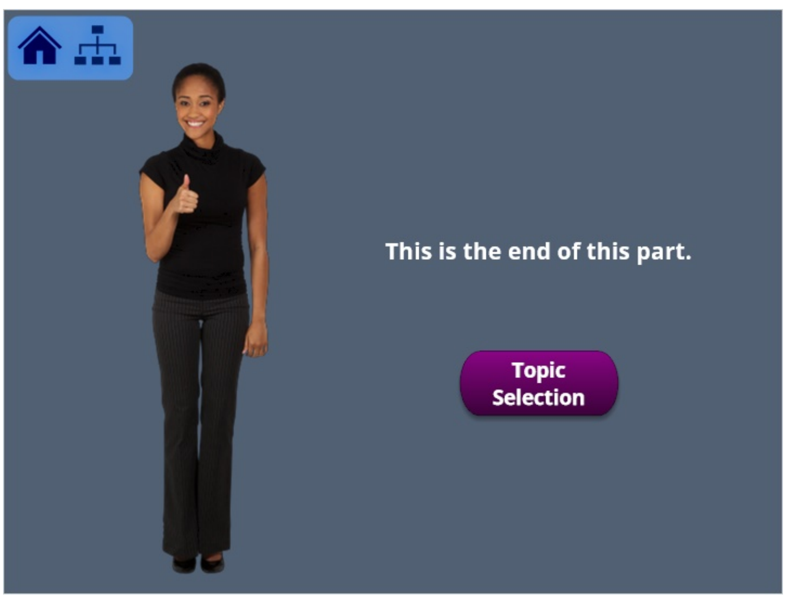
4.19 Parameter Matrix (III)




```
data all_parms;
    set person_parm;
    If _N_=1 then set items_wide ;
run;
```

	theta	a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11	a12	a13	a14	b1	b2	b3
1	0.3877363395	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
2	-0.406452884	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
3	1.3881172731	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
4	0.9263682846	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
5	-1.365842258	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
6	-1.435905431	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
7	1.1636979787	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
8	0.4595632147	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
9	-0.436601153	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
10	-0.259673442	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
11	0.612646981	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
12	-1.101398549	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
13	-1.390194134	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
14	-0.506521292	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
15	-0.899980914	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
16	0.2915990242	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
17	0.49479963	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5
18	0.5957883485	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	1.9	1.9	1.9	1.9	-2	-1	-0.5

4.20 Bookend: Step 2



4.21 Score Computation (I)

Response Probabilities

We want a data file that looks like:

Item 1	Item 2	Item 3	...	Item 14
1	1	0	...	1
0	1	1	...	0
⋮	⋮	⋮	⋱	⋮
0	1	1	...	0

Let's consider one individual:

Theta	a_1	b_1	...	a_{14}	b_{14}
-1.43	1.7	-2	...	1.9	2

$$P_j(U_j = 1|\boldsymbol{\omega}_j, \theta) = \frac{1}{1 + e^{-a_j(\theta - b_j)}}$$

For item 1: $P(U_1 = 1) = \frac{1}{1+e^{-1.7(-1.43-(-2))}} = .725$

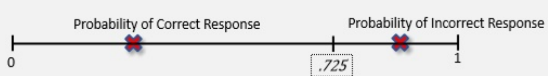
⋮

For item 14: $P(U_{14} = 1) = \frac{1}{1+e^{-1.9(-1.43-(2))}} = .0015$

4.22 Score Computation (II)

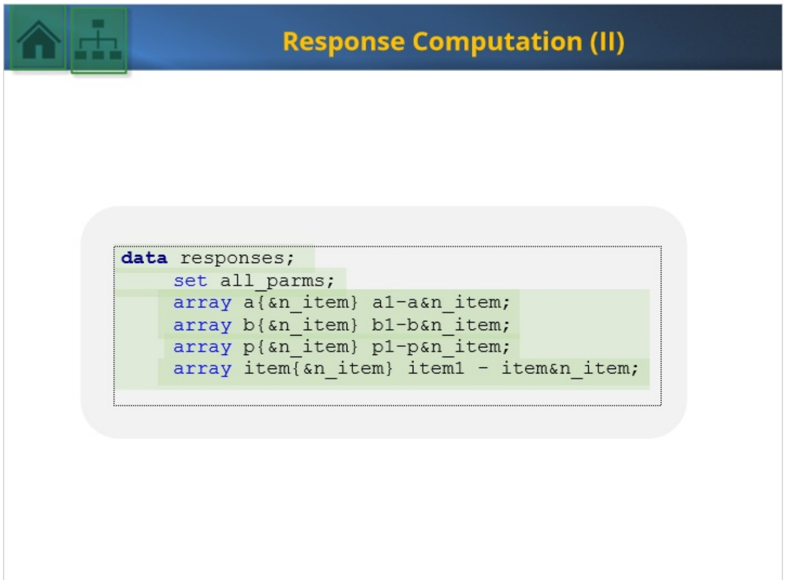
Response Computation (I)

- Incorporate **randomness** into the process
- Choose a **random number** between 0 and 1 with **equal probability** from a **uniform distribution**
  - ✓ If the randomly selected value is **less than or equal to .725**, assign a score of **1**
  - ✓ If the randomly selected value is **greater than .725**, assign a score of **0**

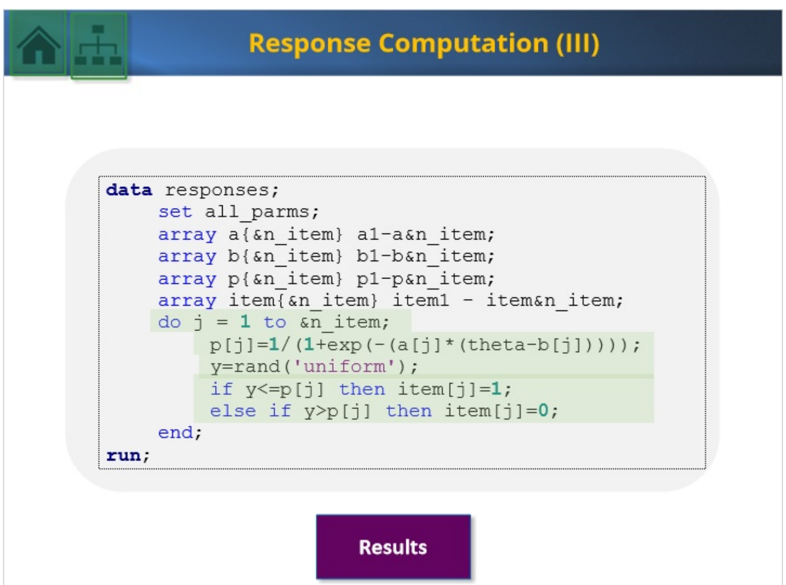


For item 1: $P(U_1 = 1) = \frac{1}{1+e^{-1.7(-1.43-(-2))}} = .725$

4.23 Score Computation (III)



4.24 Score Computation (IV)



Results (Slide Layer)

Results

Obs	p1	p2	p3	p4	p5	p6	p7	item1	item2	item3	item4	item5	item6	item7
20	0.96695	0.84241	0.69557	0.49407	0.29448	0.15139	0.03156	1	1	1	0	1	1	0
21	0.90752	0.64193	0.43382	0.24671	0.12279	0.05645	0.01081	0	1	0	0	0	0	0
22	0.74623	0.34946	0.18673	0.08937	0.04026	0.01761	0.00326	1	1	0	1	1	0	0
23	0.99891	0.99404	0.98616	0.96821	0.92866	0.84764	0.50406	1	1	1	1	1	1	1
24	0.98711	0.93327	0.85669	0.71870	0.52200	0.31822	0.07857	1	1	1	1	0	0	1
25	0.90371	0.63161	0.42290	0.23851	0.11807	0.05412	0.01034	1	1	0	1	0	0	0

Back

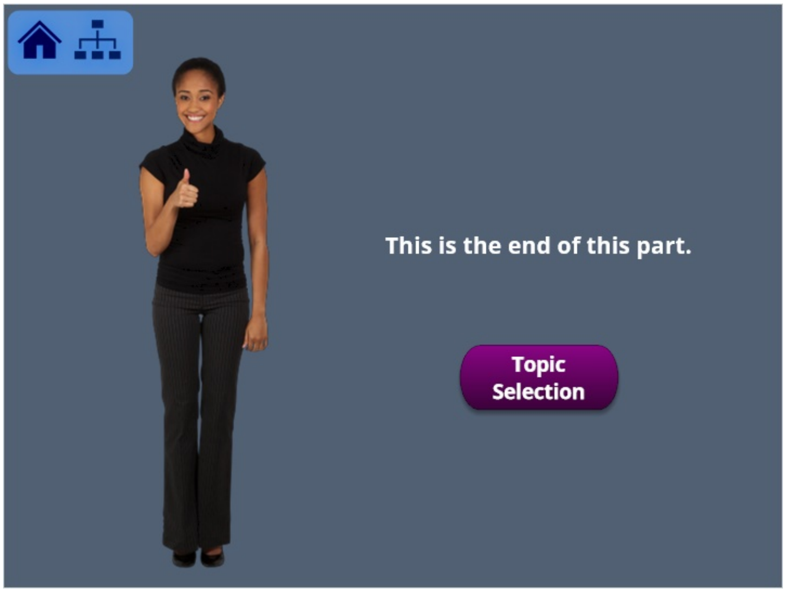
4.25 Score Computation (V)

Data Set Cleanup

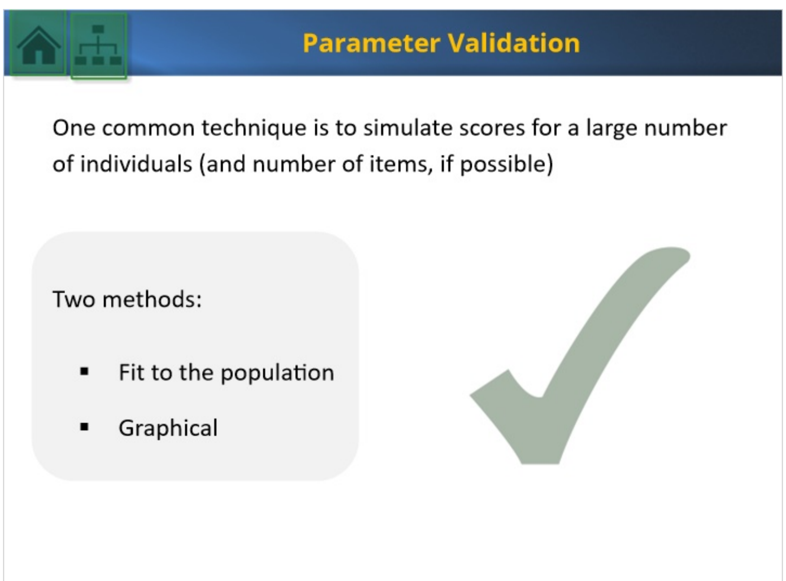
```
data responses_only;
     set responses;
     keep item:;
run;
```

	item1	item2	item3	item4	item5	item6	item7	item8	item9	item10
1	1	1	1	1	0	0	0	1	1	1
2	1	0	1	0	1	0	0	1	0	1
3	1	1	1	1	0	0	0	1	1	0
4	1	1	1	1	0	0	0	1	1	1
5	1	1	1	1	0	0	0	1	1	1
6	1	0	0	0	0	0	0	1	1	0
7	1	1	1	1	1	1	0	1	0	1
8	1	1	1	1	1	0	0	1	1	1
9	1	1	1	1	1	0	0	1	0	1
10	1	1	1	1	1	1	0	1	1	0
11	0	0	0	0	0	0	0	0	1	0
12	1	1	0	1	0	1	0	1	1	0
13	1	1	1	1	1	1	0	1	1	1
14	1	1	1	1	1	0	0	1	1	1
15	1	1	1	1	1	0	0	1	1	1
16	1	1	1	1	0	0	0	1	0	1

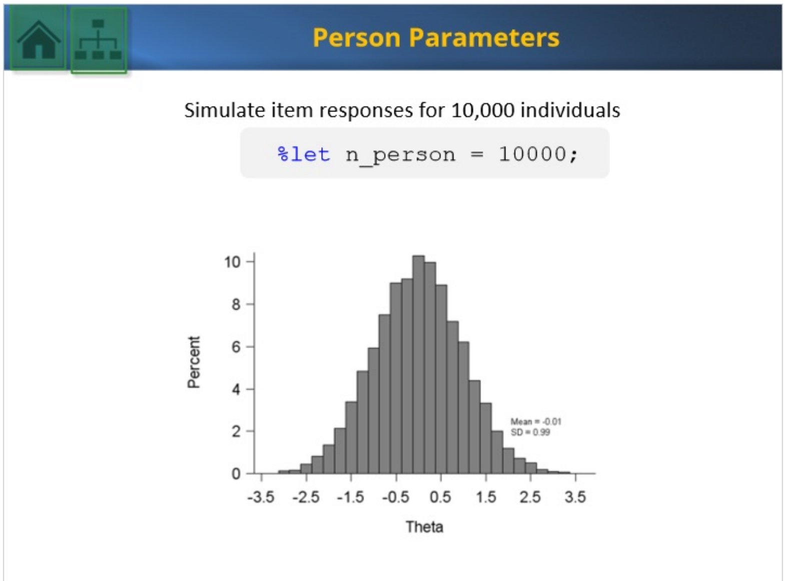
4.26 Bookend: Step 3



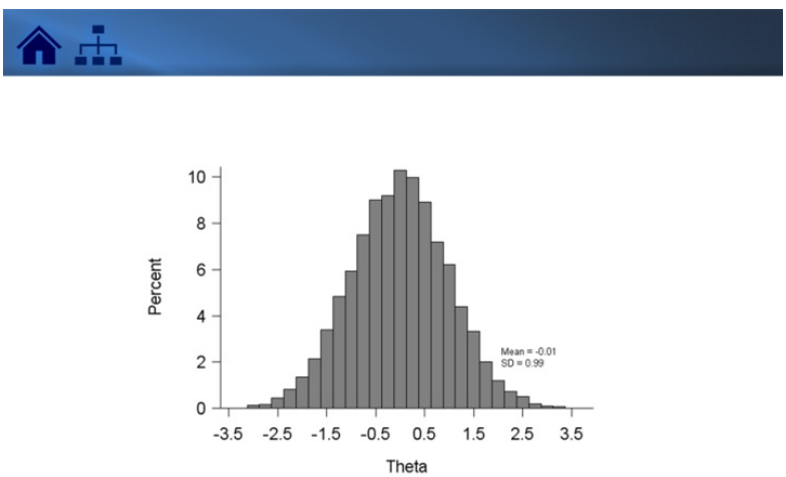
4.27 Parameter Validation (I)



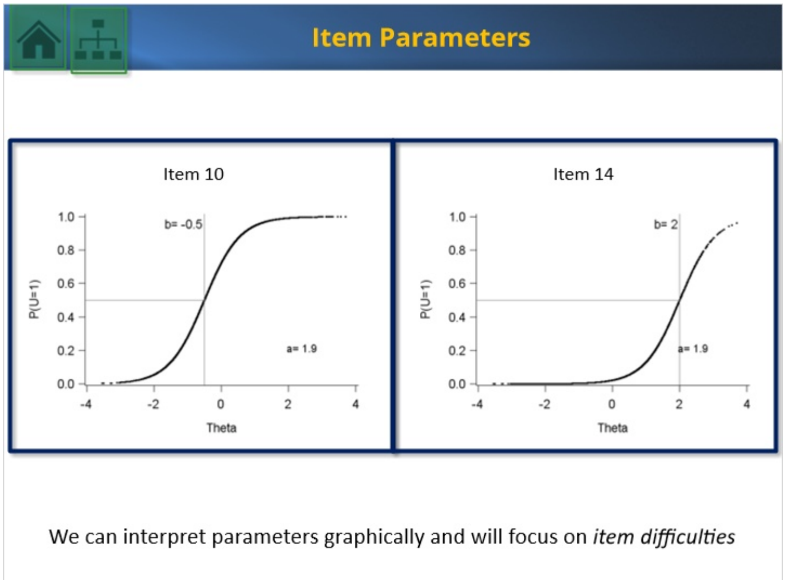
4.28 Parameter Validation (II)



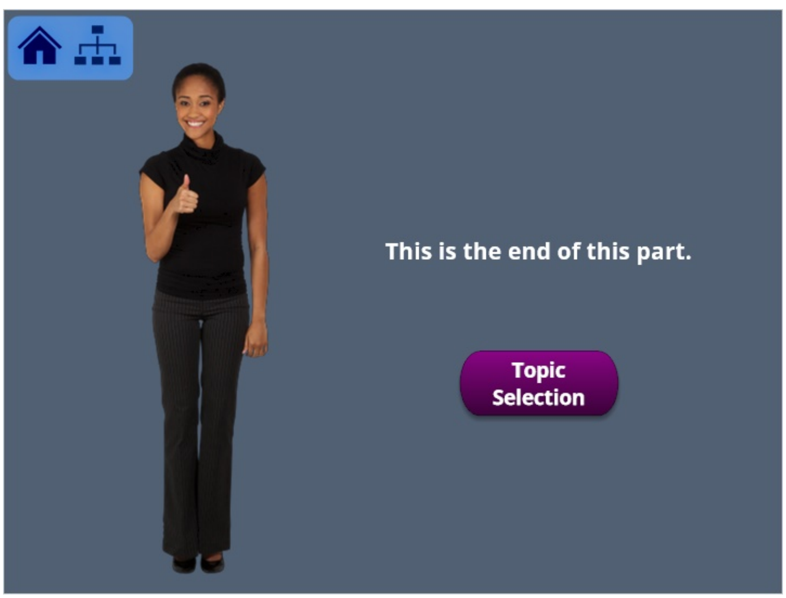
Results (Slide Layer)



4.29 Parameter Validation (III)



4.30 Bookend: Step 4



4.31 Replications (I)

Replications

"*within each condition*, there are often a certain number of replications. That is, for many research questions, it may be necessary to repeat the data generation process over multiple occasions—that is, replications—so that the empirical estimate of the sampling distribution (due to the simulation) of various statistics of interest may be observed"

Feinberg and Rubright, 2016

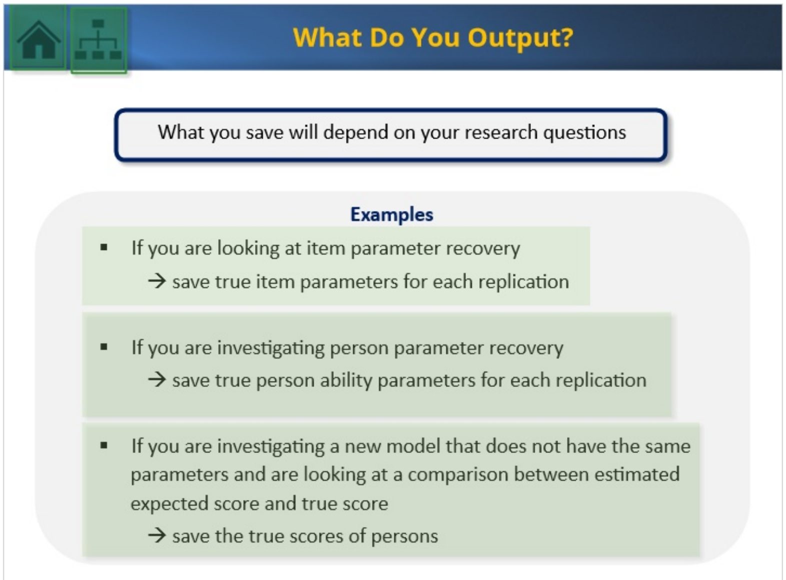
4.32 Replications (II)

Replications

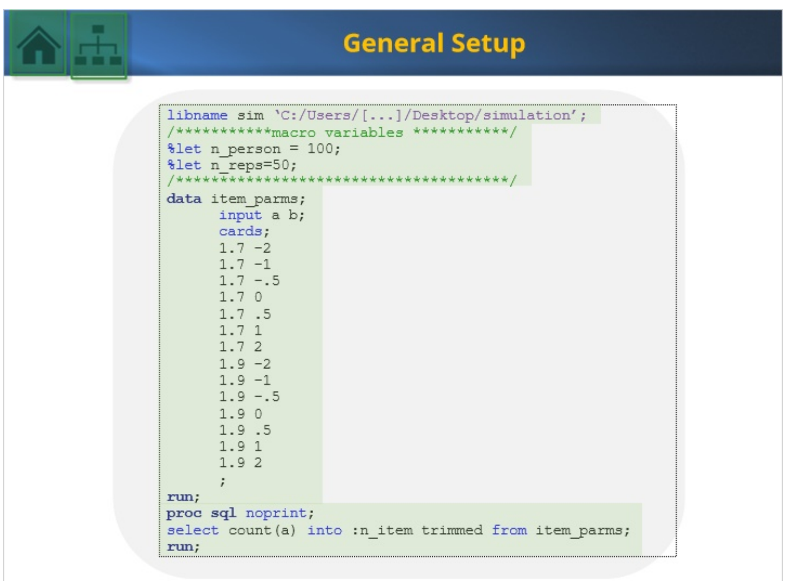
```
%let n_reps=50;
%macro repeatIRTgen;
    %do rep = 1 %to &n_reps;
        [data generation commands]
    %end;
%mend repeatIRTgen;
%repeatIRTgen;
```

```
libname sim 'C:\Users\desktop\simulation';
data sim.Data&rep;
    set responses;
    keep item: theta;
run;
```

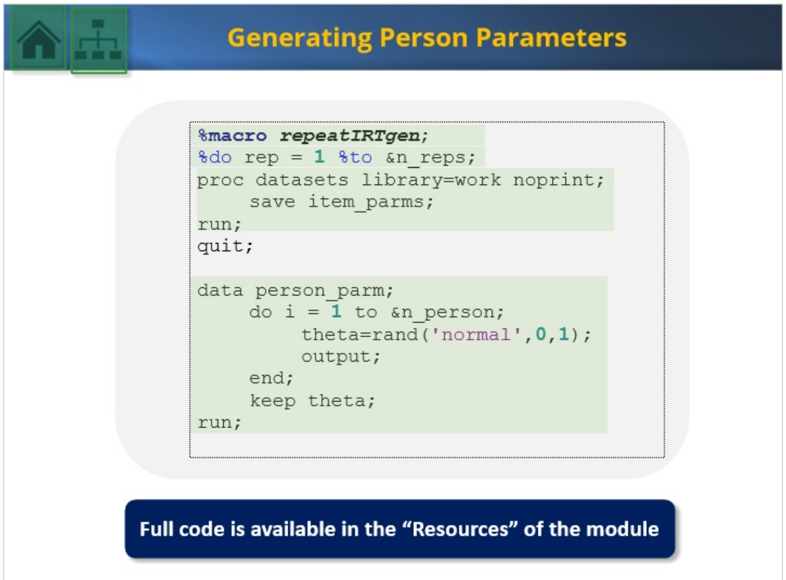
4.33 Replications (III)



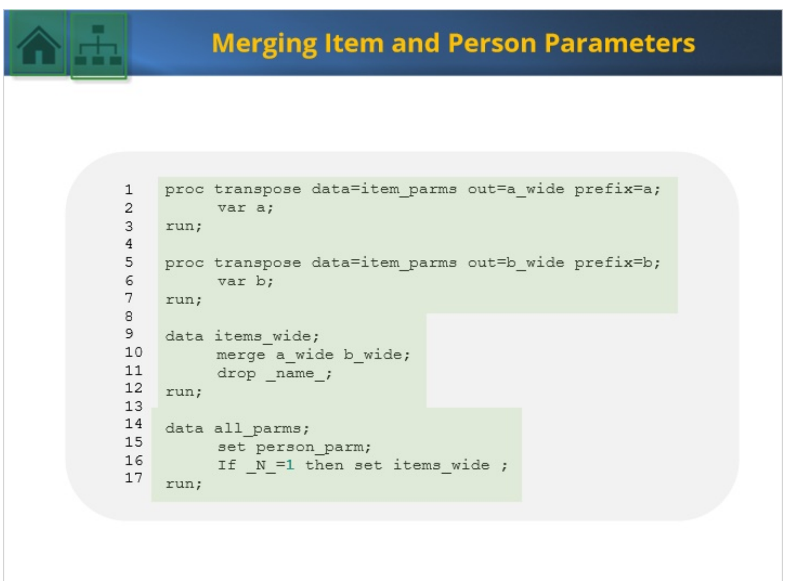
4.34 Replications (IV)



4.35 Replications (V)



4.36 Replications (VI)



4.37 Replications (VII)

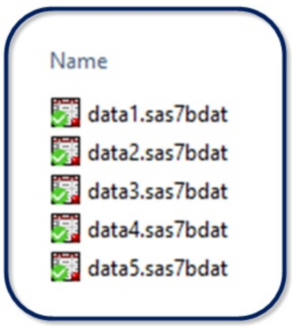
Simulating Responses

```
data responses;
     set all_parms;
     array a{&n_item} a1-a&n_item;
     array b{&n_item} b1-b&n_item;
     array p{&n_item} p1-p&n_item;
     array item{&n_item} item1 - item&n_item;
     do j = 1 to &n_item;
         p[j]=1/(1+exp(-(a[j]*(theta-b[j]))));
         y=rand('uniform');
         if y<=p[j] then item[j]=1;
         else if y>p[j] then item[j]=0;
     end;
run;
```

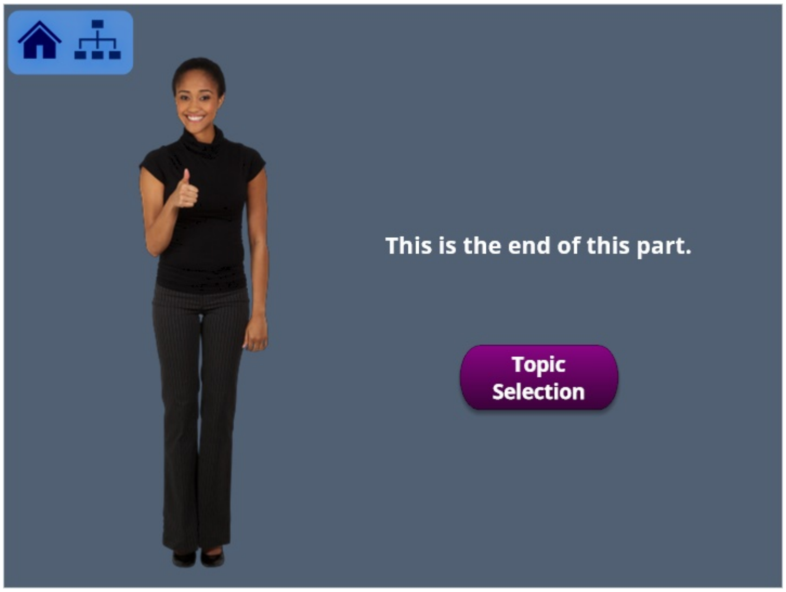
4.38 Replications (VIII)

Saving Generated Data Sets

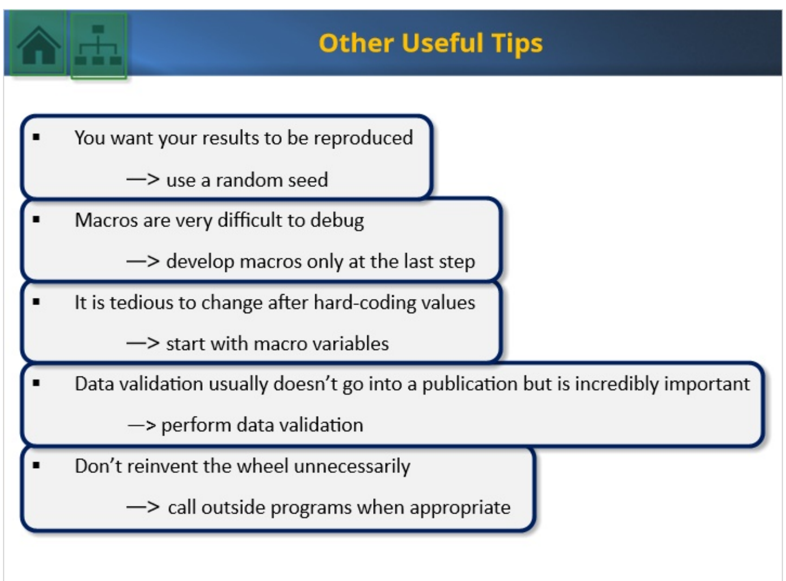
```
data sim.Data&rep;
     set responses;
     keep item: theta;
run;

%end;
%mend repeatIRTgen;


%repeatIRTgen;
```



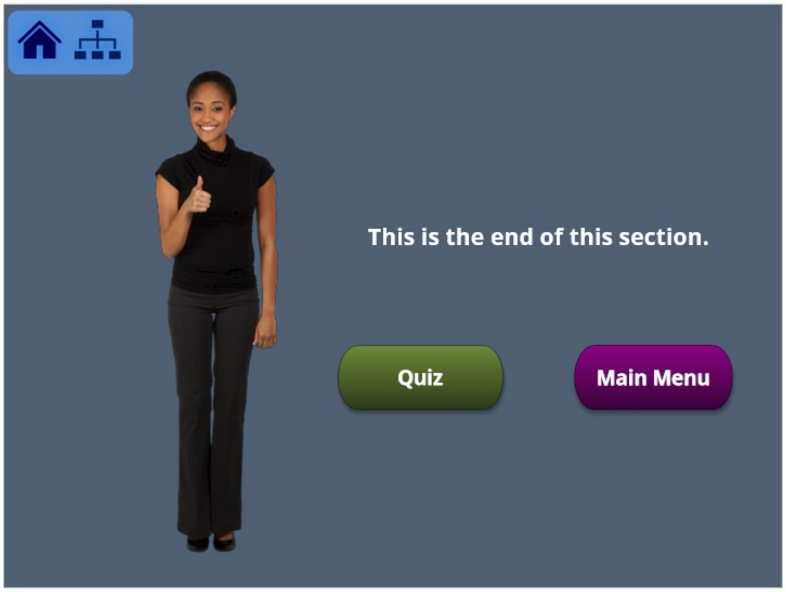
4.39 Bookend: Step 5



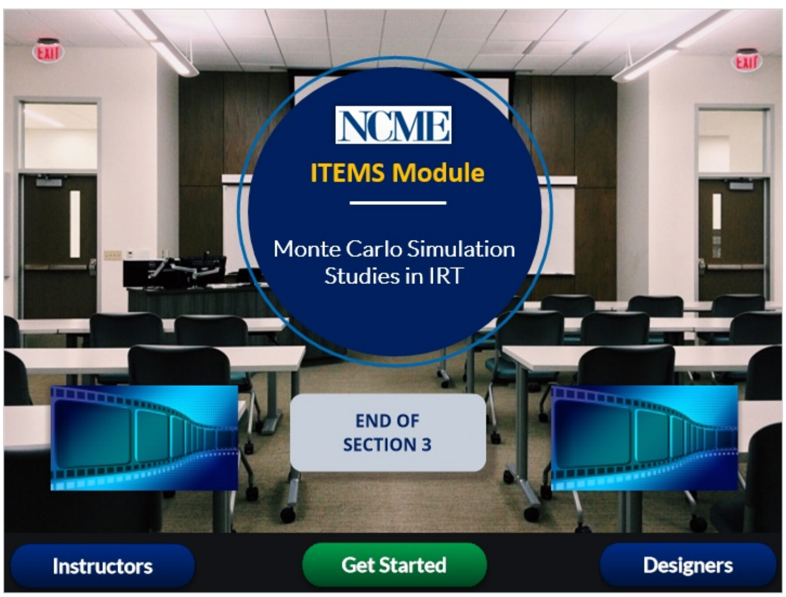
4.40 Other Useful Tips



4.41 Bookend: Section 3

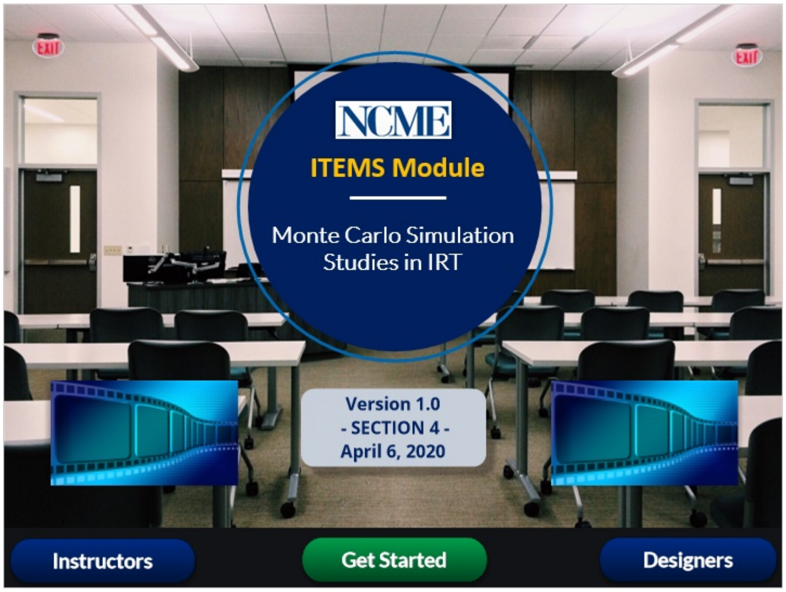


4.42 Module Cover (END)

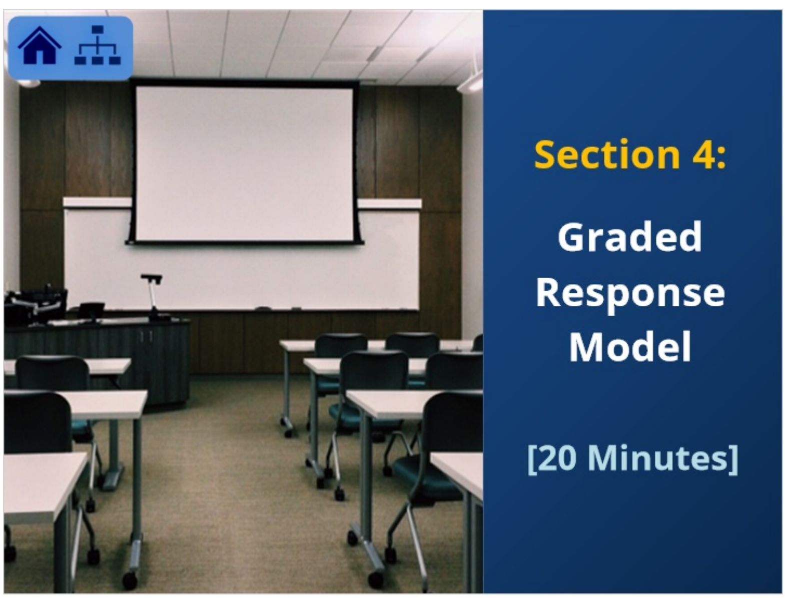


5. Section 4: Graded Response Model

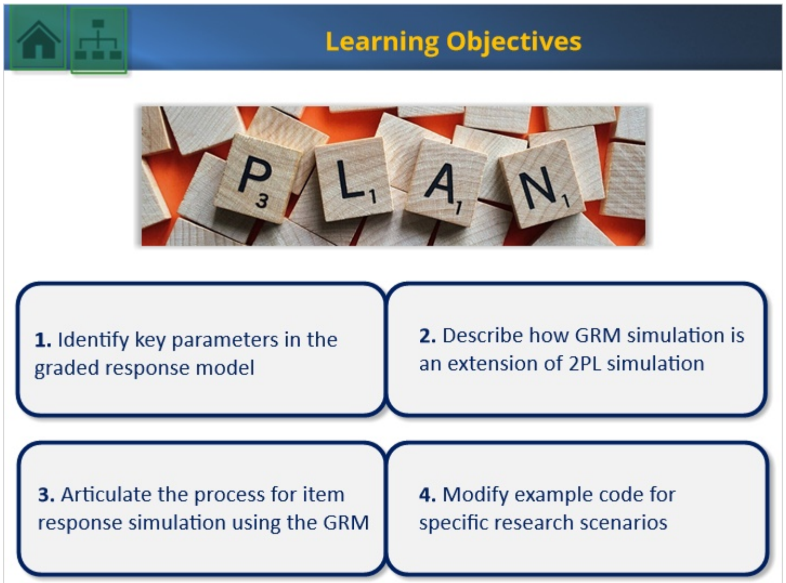
5.1 Module Cover (START)



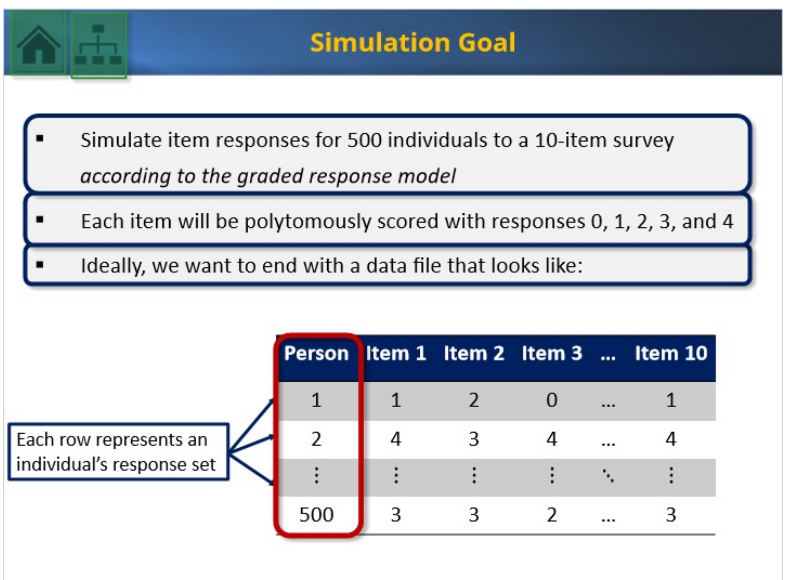
5.2 Cover: Section 4



5.3 Objectives



5.4 Simulation Goal



Person	Item 1	Item 2	Item 3	...	Item 10
1	1	2	0	...	1
2	4	3	4	...	4
⋮	⋮	⋮	⋮	⋱	⋮
500	3	3	2	...	3

5.5 Topic Selection

5.6 Bookmark: GRM

5.7 GRM (I)

Overview

Probabilities (p_{ijk}) for individual score categories are defined as differences between cumulative probabilities (p^*_{ijk})

Example

$$p^*_{ijk=3} - p^*_{ijk=4} = p_{ijk=3}$$

1 2 3 4 5 – 1 2 3 4 5 = 1 2 3 4 5

Each p^*_{ijk} is modeled as a 2PL

5.8 GRM (II)

Core Formula

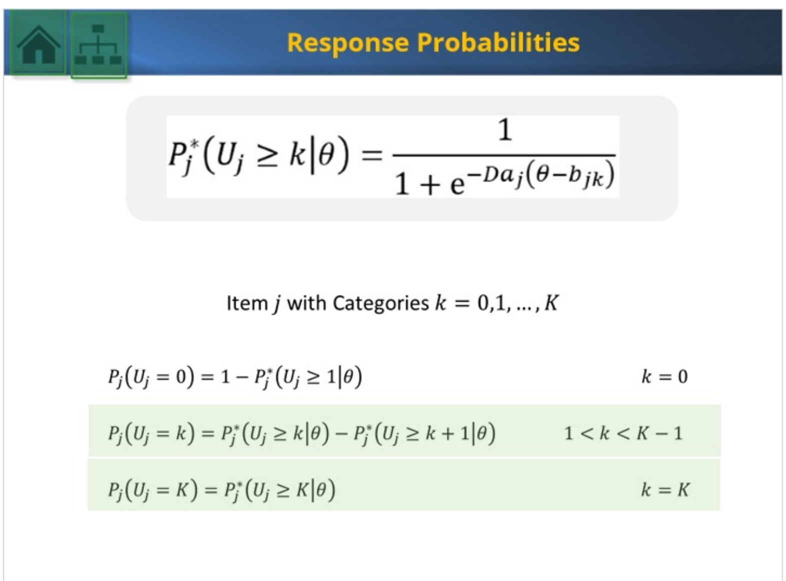
$$P^*_j(U_j \geq k|\theta) = \frac{1}{1 + e^{-Da_j(\theta - b_{jk})}}$$

- Cumulative probability an individual with a given ability or trait θ selects or receives a category score of k or higher for item j
- U_j: response on item j
- θ is an ability value on the trait of interest
- a_j is the slope/discrimination parameter for item j
- b_{jk} is the threshold parameter for item j and category k
- D is the scaling parameter
  - ✓ Here we set $D = 1$
  - ✓ Other common value is $D = 1.702$

Reference

References (Slide Layer)

5.9 GRM (III)

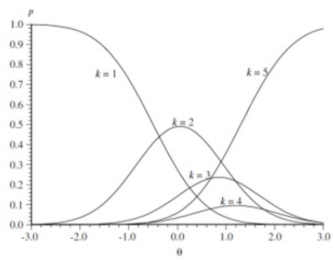


5.10 GRM (IV)

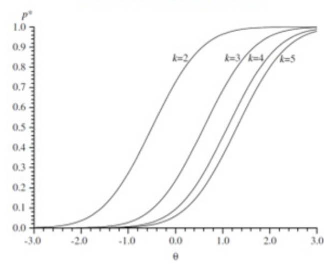
Example: $\theta = 1$

k	p_{ijk}	p^*_{ijk}
1	.036	1
2	.280	.964
3	.232	.684
4	.093	.452
5	.359	.359

Category Probabilities



Cumulative Probabilities



5.11 GRM (V)

Thresholds

- b_{jk} is the threshold parameter for item j and category k
- b_{jk} is the point on the θ scale in which a respondent has a 0.5 probability of selecting that category or greater
- For an item with K categories, there are $\mathrm{K} - 1$ thresholds

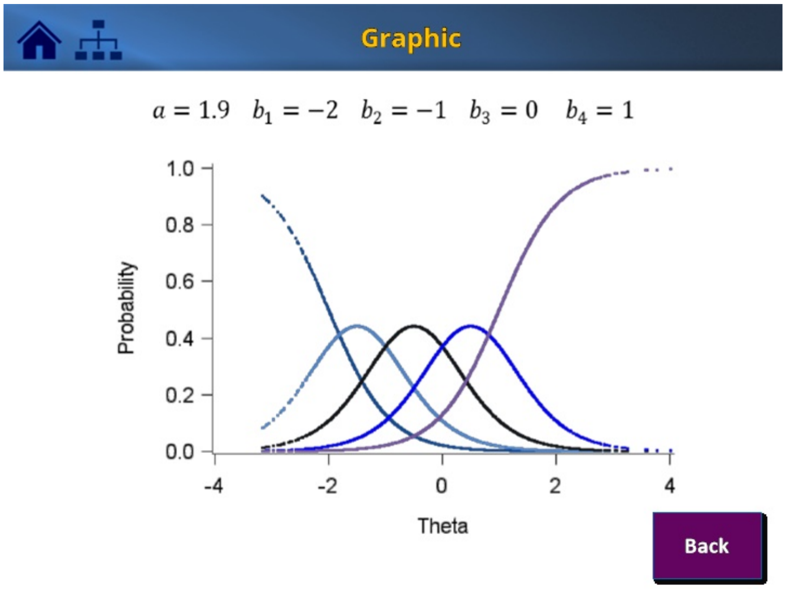
Example

For an item with 5 categories scored 0, 1, 2, 3, and 4, there are four thresholds such that:

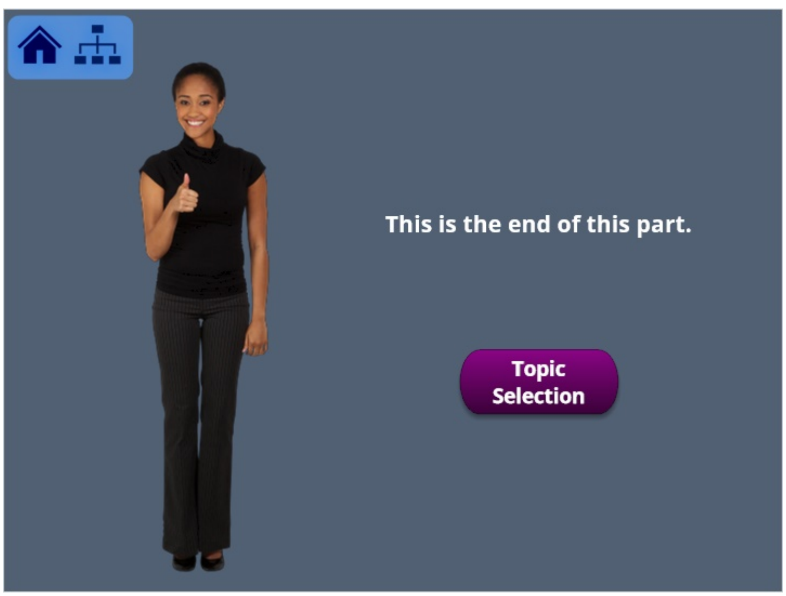
$$b_{j1} \leq b_{j2} \leq b_{j3} \leq b_{j4}$$

Graphic

Graphic (Slide Layer)

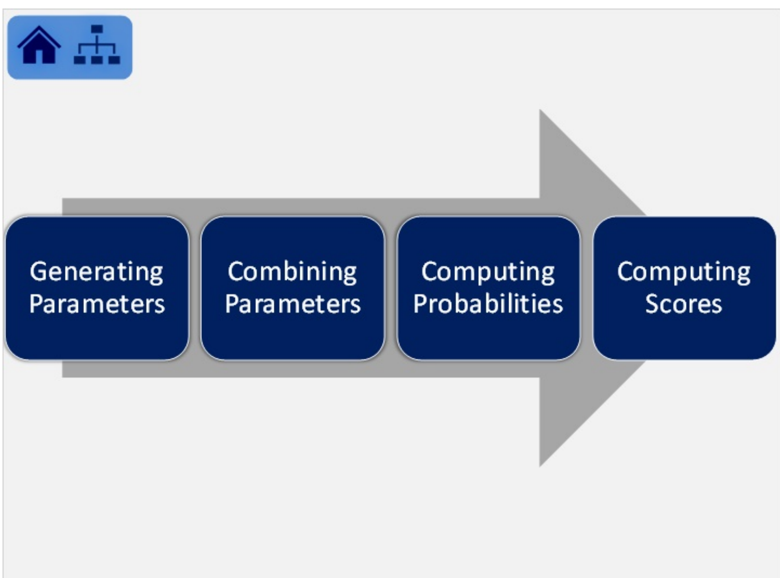


5.12 Bookend: GRM



5.13 Bookmark: Data Simulation

5.14 Step Selection



5.15 Person Parameters

Person Parameters

```
%let n_person =500;
data person_parm;
  do i = 1 to &n_person;
    theta=rand('normal',0,1);
    output;
  end;
  keep theta;
run;
```

Create a value named *theta*. Randomly select theta value from a normal distribution with mean = 0 and var = 1

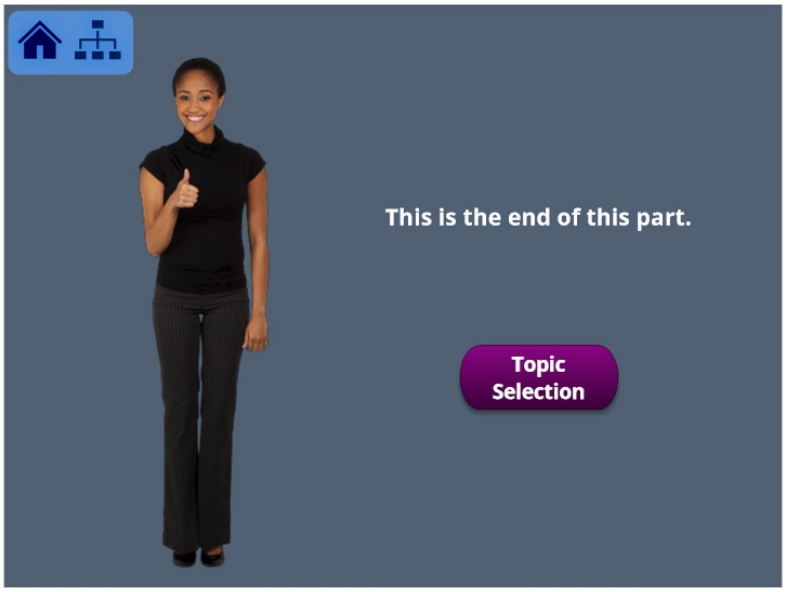
5.16 Item Parameters

Item Parameters

- Create a data set that contains **item parameters**
- This may be based off of **empirical estimates**, **previous literature, etc.**
- You may **randomly select** item parameters as well
- **Literature** can be helpful in determining what **distribution** to randomly select from

```
data item_parms;
    input a b1 b2 b3 b4;
    cards;
    1.2    -2    -1    0    1
    1.2  -1.5   -.5   .5  1.5
    1.2    -1     0    1    2
    1.2   -.5    .5  1.5  2.5
    1.2     0    .5    1  1.5
    1.9    -2    -1    0    1
    1.9  -1.5   -.5   .5  1.5
    1.9    -1     0    1    2
    1.9   -.5    .5  1.5  2.5
    1.9     0    .5    1  1.5
    ;
run;
```

5.17 Bookend: Step 1



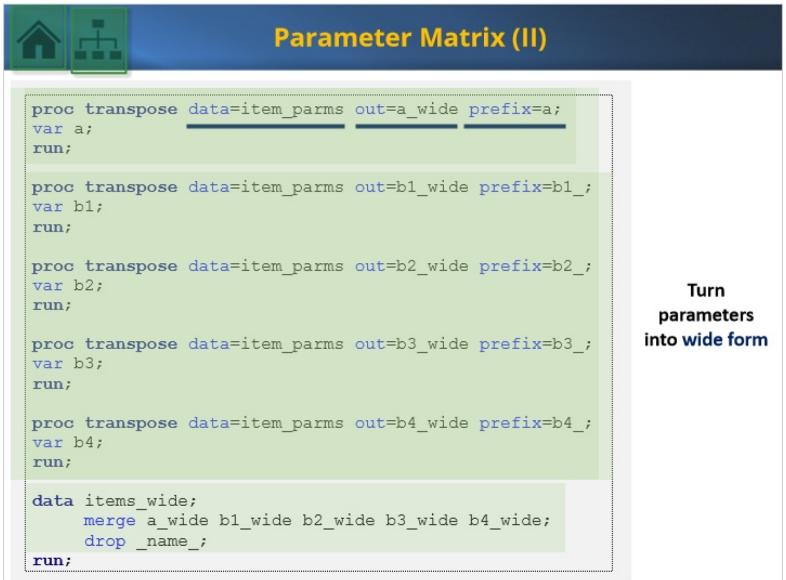
5.18 Parameter Matrix (I)

Parameter Matrix (I)

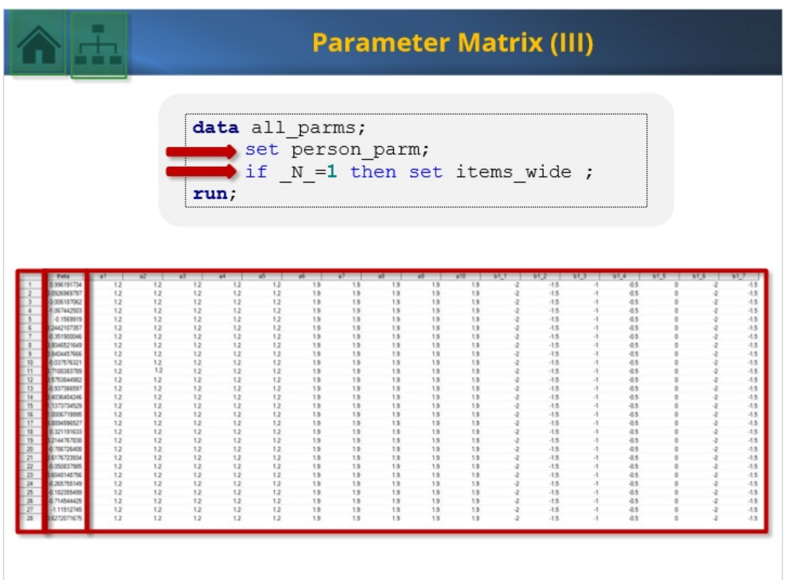
- We need in the same data set / parameter matrix:
  - ✓ an individual's ability parameter θ
  - ✓ item parameters a_j and b_{jk}
- We want our data set / parameter matrix to look like this:

Theta	a_1	$b_{1,1}$	$b_{1,2}$	$b_{1,3}$	$b_{1,4}$	...	a_{10}	$b_{10,1}$	$b_{10,2}$	$b_{10,3}$	$b_{10,4}$
1.43	1.2	-2	-1	0	1	...	1.9	0	.5	1	1.5
2.81	1.2	-2	-1	0	1	...	1.9	0	.5	1	1.5
⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋮
0.09	1.2	-2	-1	0	1	...	1.9	0	.5	1	1.5

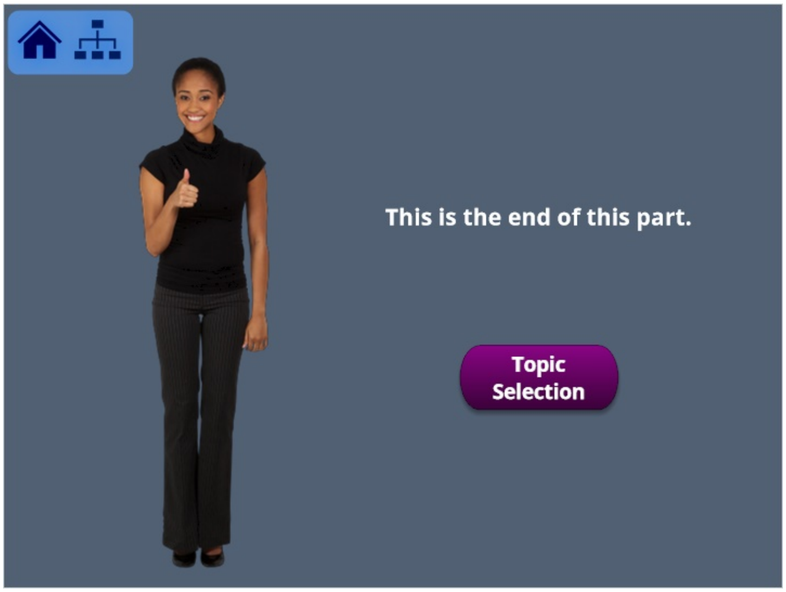
5.19 Parameter Matrix (II)



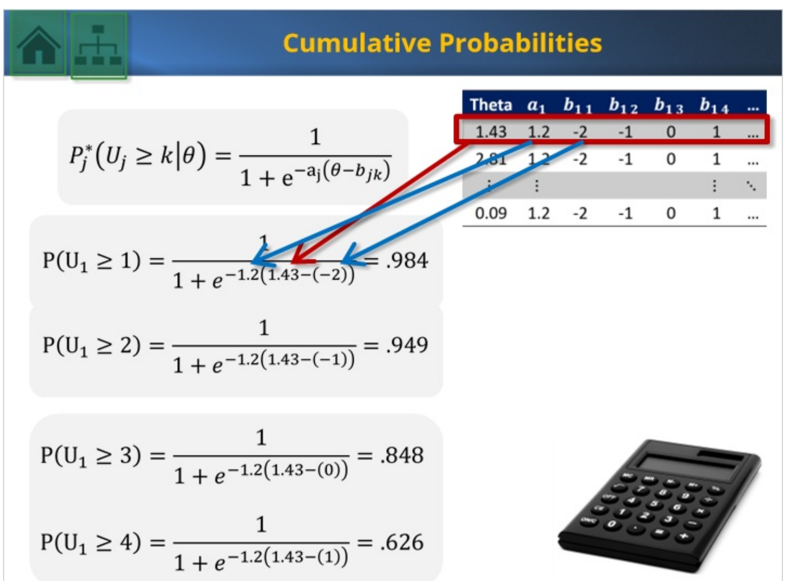
5.20 Parameter Matrix (III)



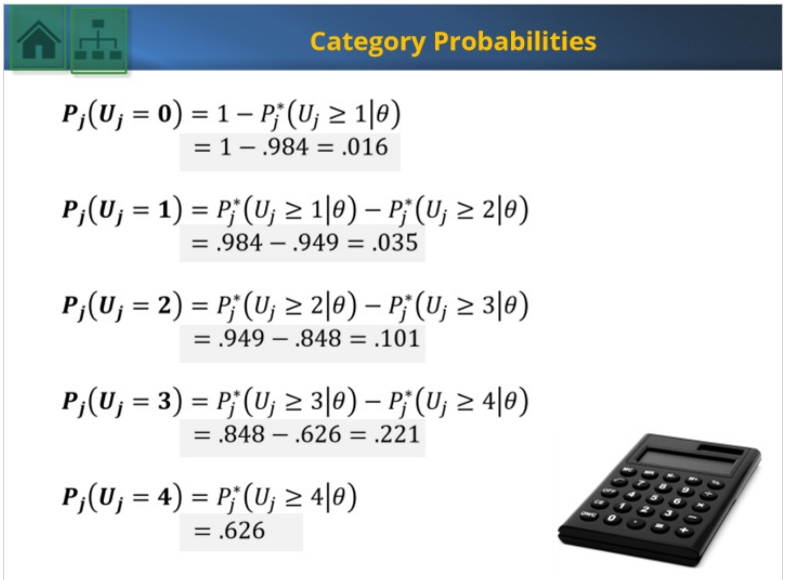
5.21 Bookend: Step 2



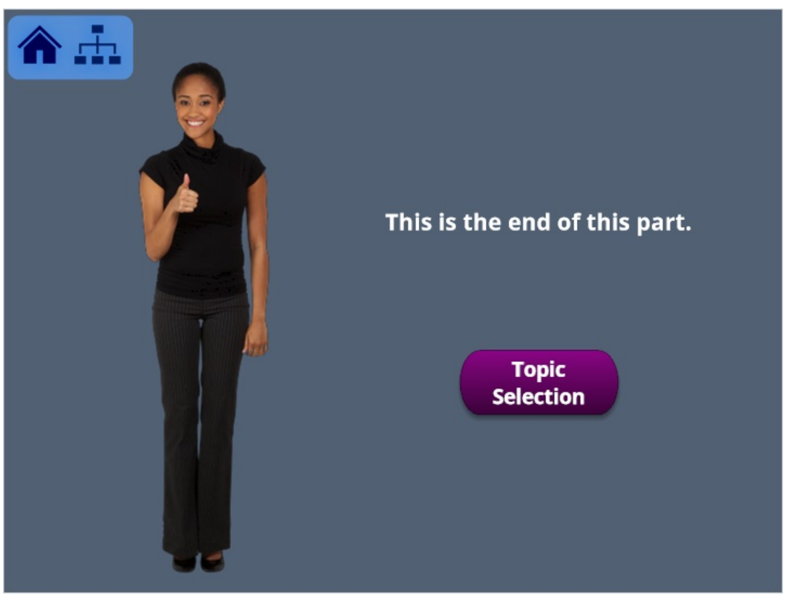
5.22 Cumulative Probabilities



5.23 Category Probabilities



5.24 Bookend: Step 3



5.25 Response Computation (I)

Response Computation (I)

- Choose a **random number** between 0 and 1 with **equal probability** from a **uniform distribution**
- If the random number falls between response categories ***k*–1** and ***k*** then response category ***k*** is assigned to the item

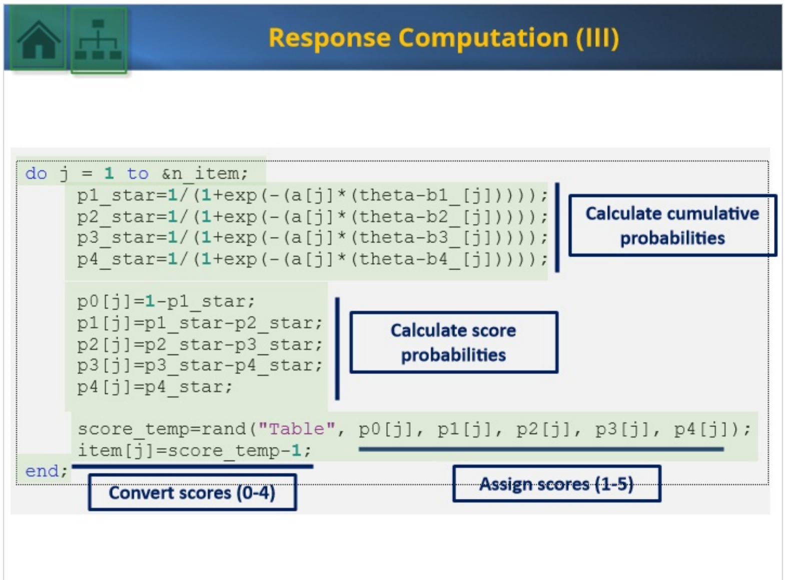
$$P_j(U_j = 0) = .016$$
$$P_j(U_j = 1) = .035$$
$$P_j(U_j = 2) = .101$$
$$P_j(U_j = 3) = .221$$
$$P_j(U_j = 4) = .626$$

5.26 Response Computation (II)

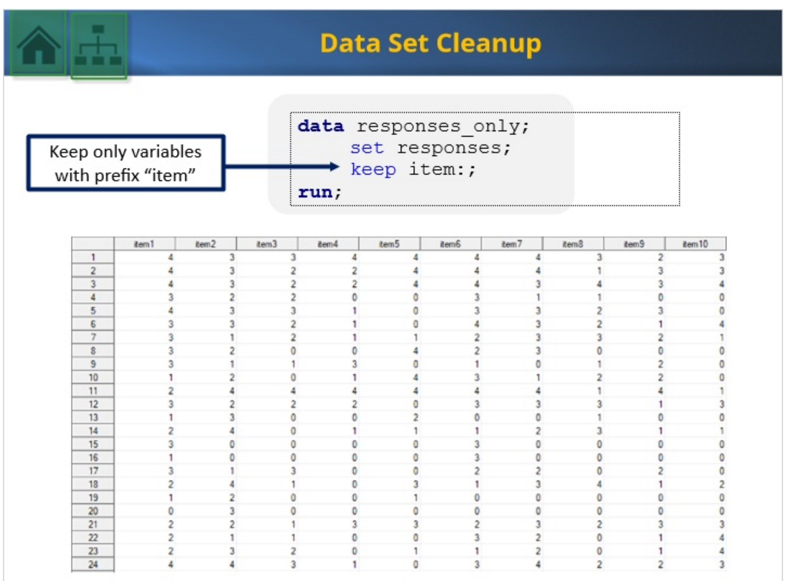
Response Computation (II)

```
data responses;
    set all_parms;
    array a{&n_item} a1-a&n_item;
    array b1_{&n_item} b1_1-b1_&n_item;
    array b2_{&n_item} b2_1-b2_&n_item;
    array b3_{&n_item} b3_1-b3_&n_item;
    array b4_{&n_item} b4_1-b4_&n_item;

    array p0{&n_item} p0_1-p0_&n_item;
    array p1{&n_item} p1_1-p1_&n_item;
    array p2{&n_item} p2_1-p2_&n_item;
    array p3{&n_item} p3_1-p3_&n_item;
    array p4{&n_item} p4_1-p4_&n_item;

    array item{&n_item} item1 - item&n_item;
```

5.27 Response Computation (III)

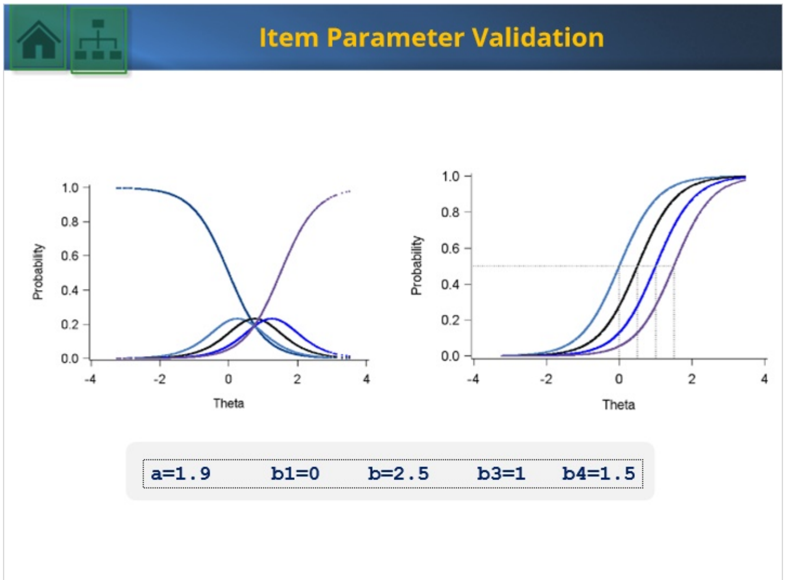


5.28 Data Set Cleanup

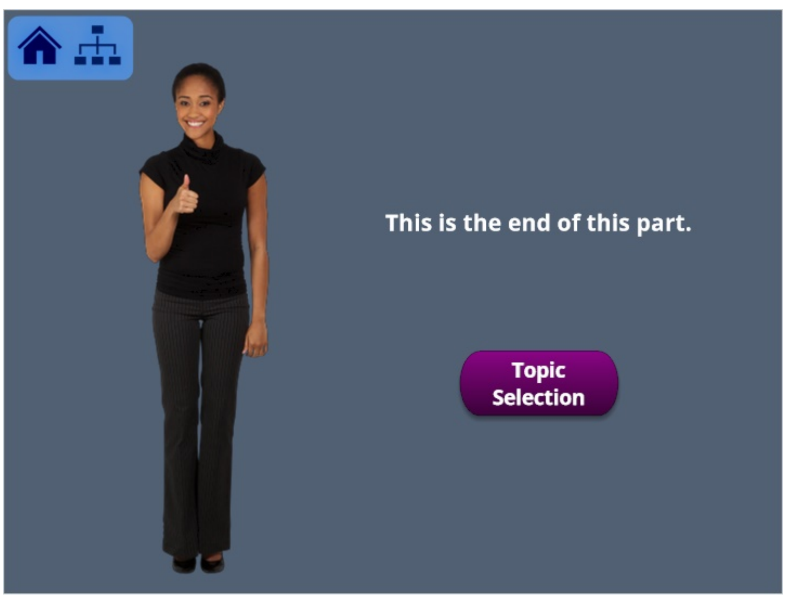


	item1	item2	item3	item4	item5	item6	item7	item8	item9	item10
1	4	3	3	4	4	4	4	3	2	3
2	4	3	2	2	4	4	4	1	3	3
3	4	3	2	2	4	4	3	4	3	4
4	3	2	2	0	0	3	1	1	0	0
5	4	3	3	1	0	3	3	2	3	0
6	3	3	2	1	0	4	3	2	1	4
7	3	1	2	1	1	2	3	3	2	1
8	3	2	0	0	4	2	3	0	0	0
9	3	1	1	3	0	1	0	1	2	0
10	1	2	0	1	4	3	1	2	2	0
11	2	4	4	4	4	4	4	1	4	1
12	3	2	2	2	0	3	3	3	1	3
13	1	3	0	0	2	0	0	1	0	0
14	2	4	0	1	1	1	2	3	1	1
15	3	0	0	0	0	3	0	0	0	0
16	1	0	0	0	0	3	0	0	0	0
17	3	1	3	0	0	2	2	0	2	0
18	2	4	1	0	3	1	3	4	1	2
19	1	2	0	0	1	0	0	0	0	0
20	0	3	0	0	0	0	0	0	0	0
21	2	2	1	3	3	2	3	2	3	3
22	2	1	1	0	0	3	2	0	1	4
23	2	3	2	0	1	1	2	0	1	4
24	4	4	3	1	0	3	4	2	2	3

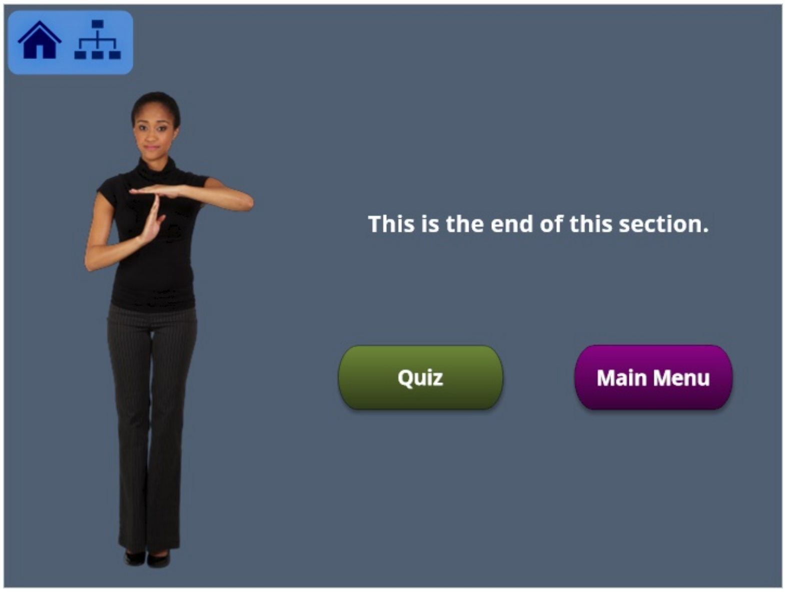
5.29 Item Parameter Validation



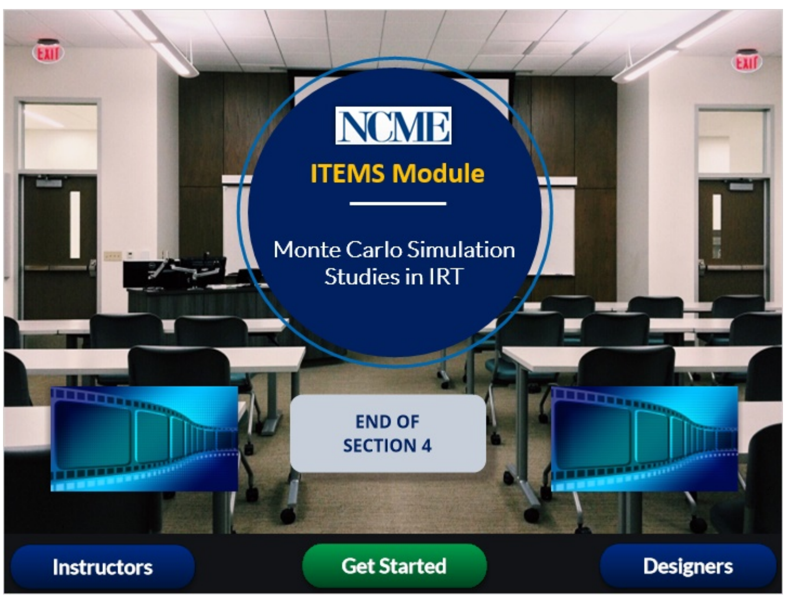
5.30 Bookend: Step 4



5.31 Bookend: Section 4

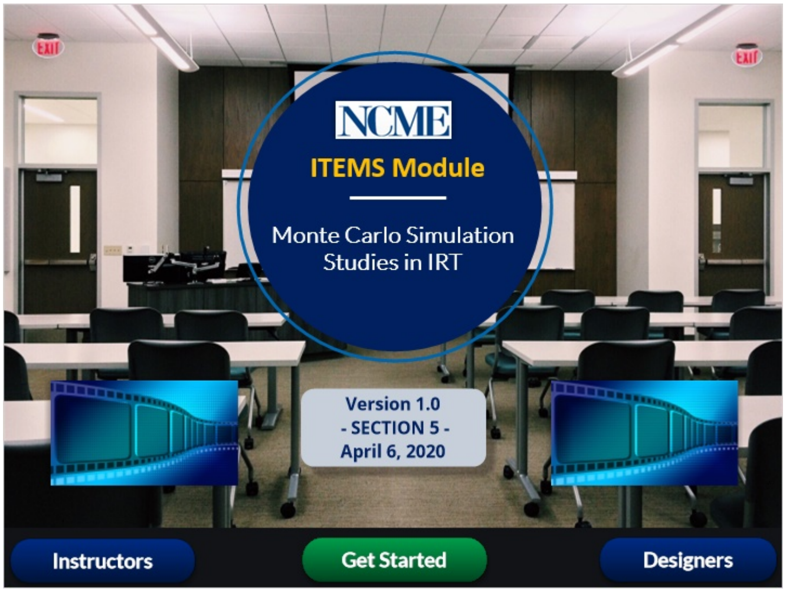


5.32 Module Cover (END)



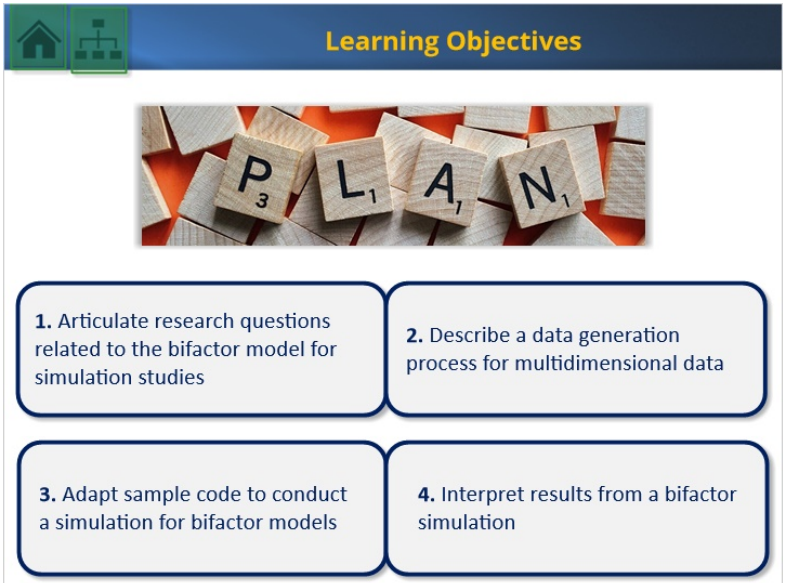
6. Section 5: Bifactor Model [Advanced]

6.1 Module Cover (START)



6.2 Cover: Section 4

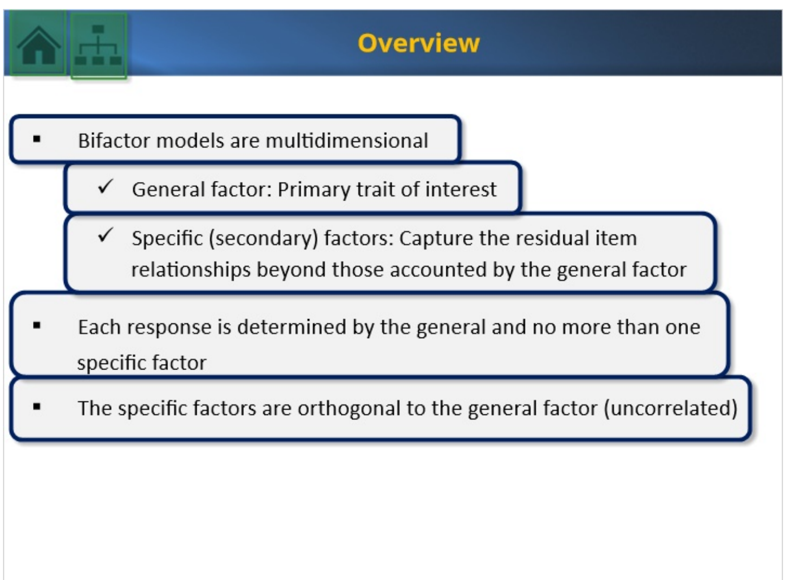
6.3 Objectives



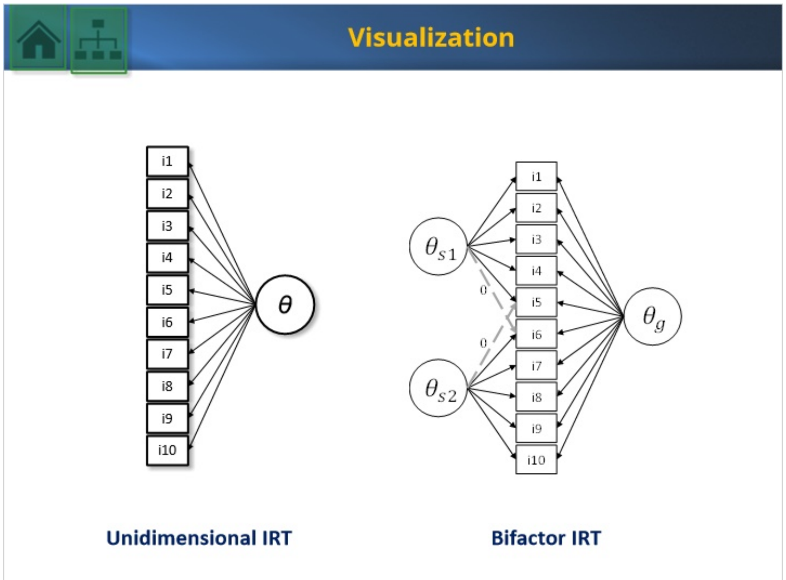
6.4 Topic Selection

6.5 Bookmark: Bifactor Model

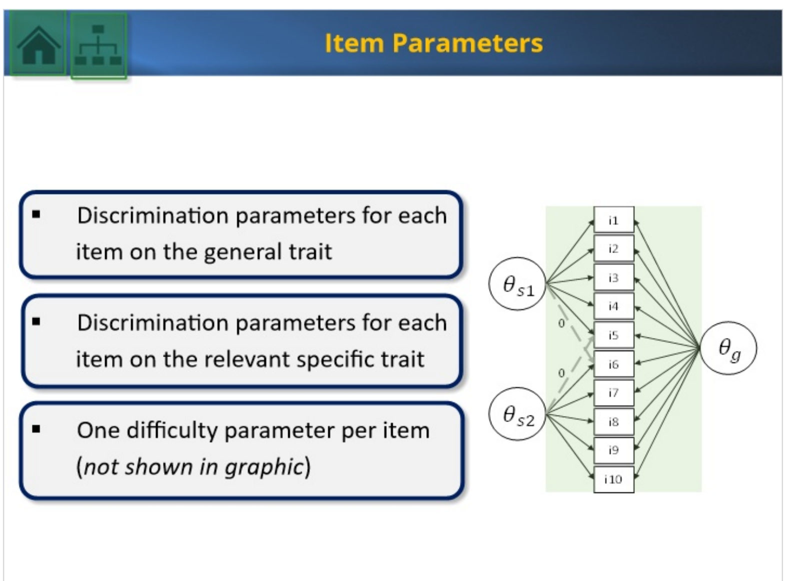
6.6 Bifactor Model (I)



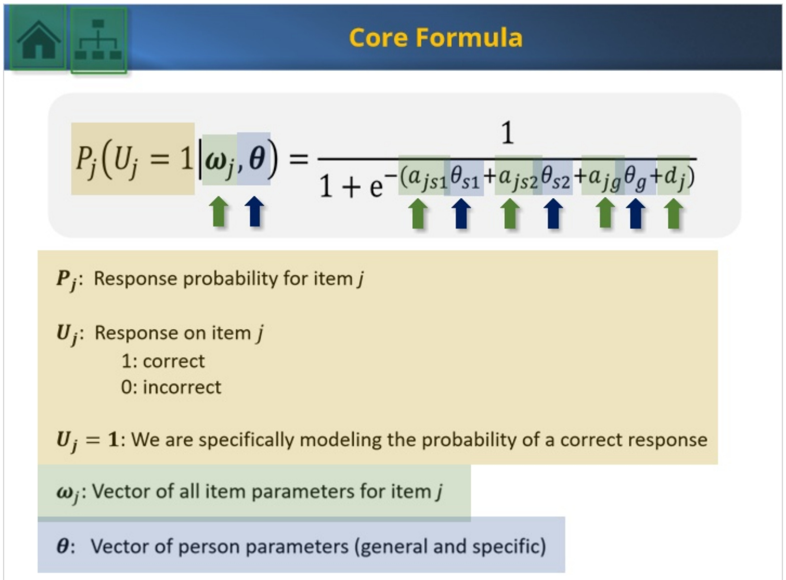
6.7 Bifactor Model (II)



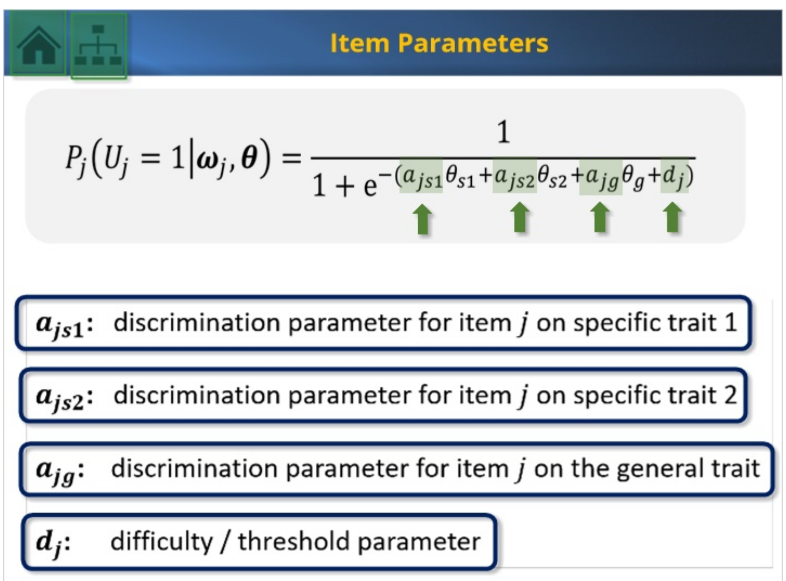
6.8 Bifactor Model (III)



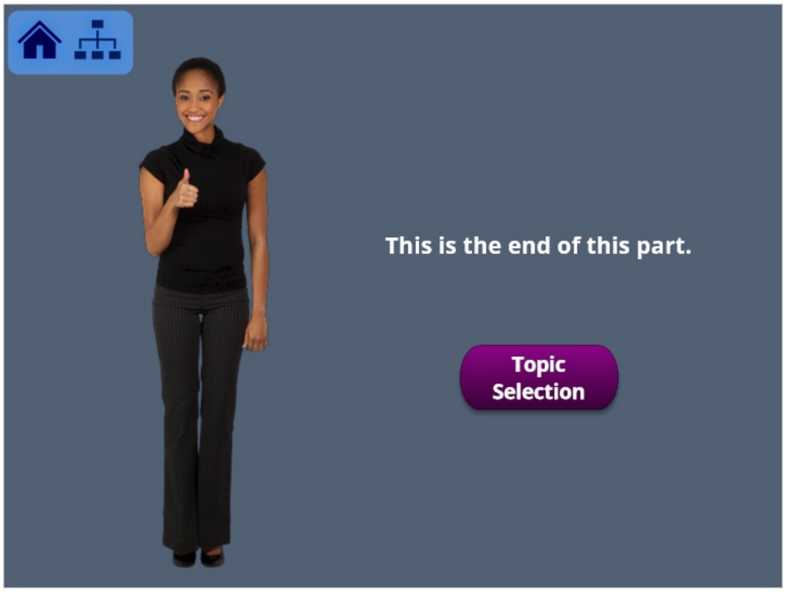
6.9 Bifactor Model (IV)



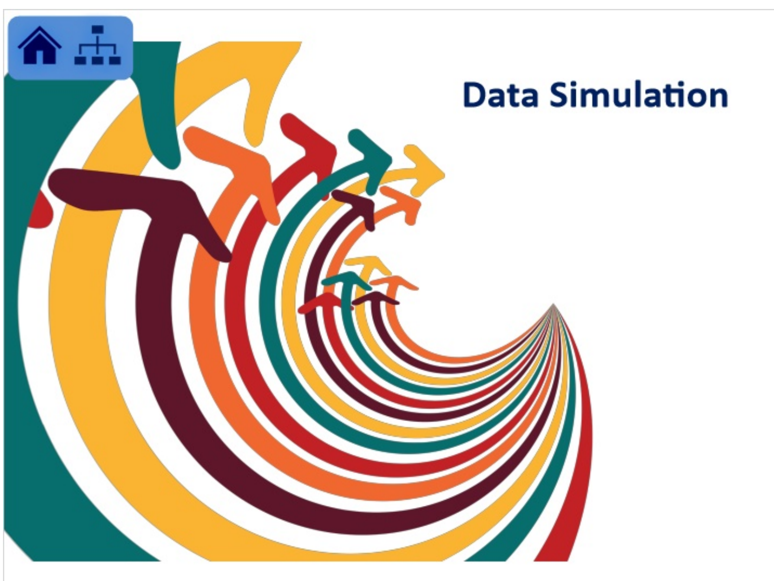
6.10 Bifactor Model (V)



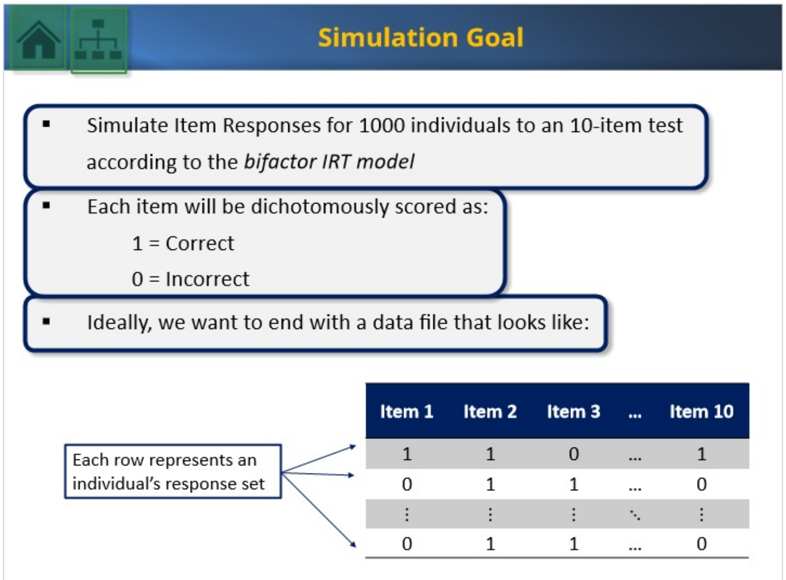
6.11 Bookend: Bifactor Model



6.12 Bookmark: Data Simulation

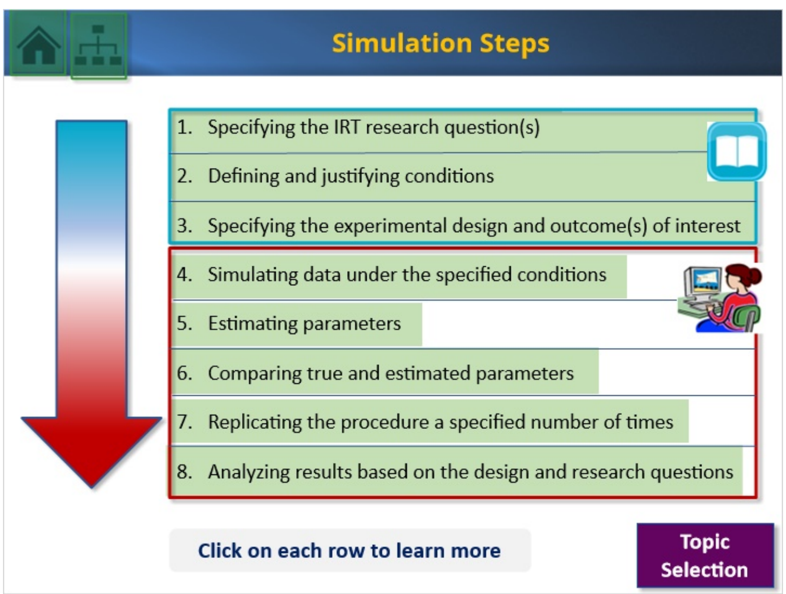


6.13 Simulation Goal

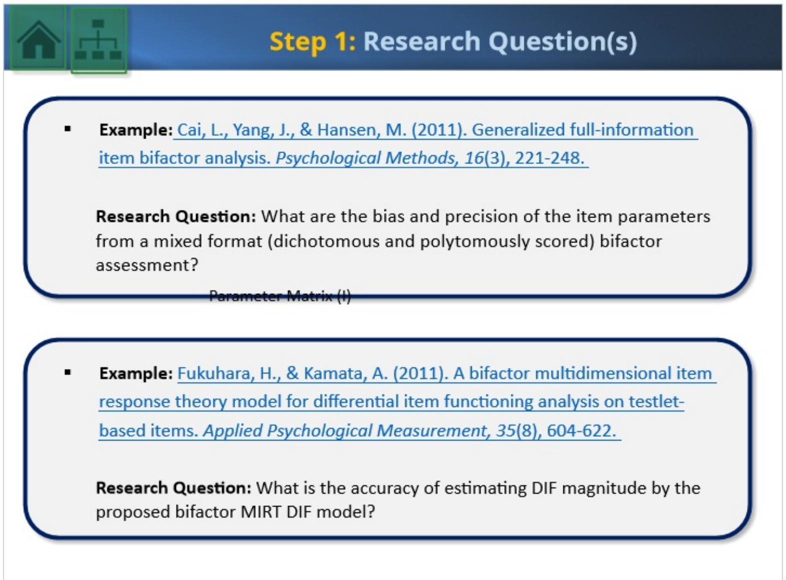


Item 1	Item 2	Item 3	...	Item 10
1	1	0	...	1
0	1	1	...	0
⋮	⋮	⋮	⋱	⋮
0	1	1	...	0

6.14 Step Selection

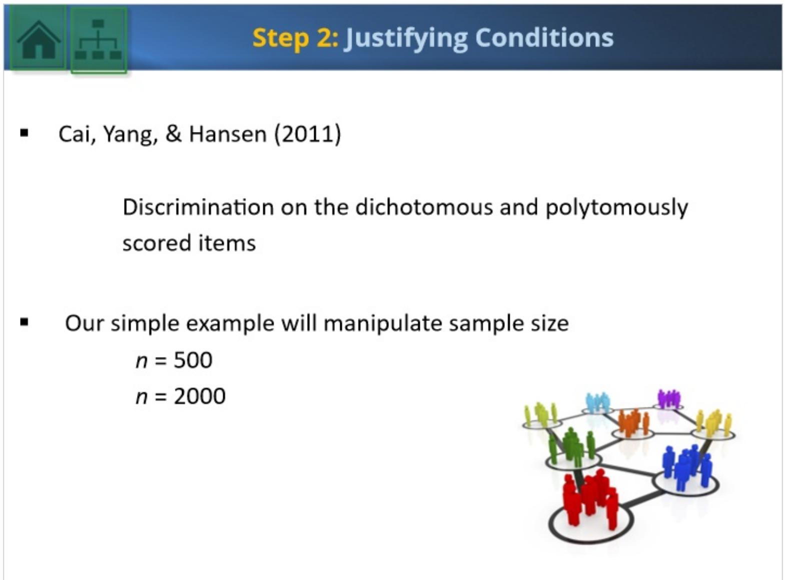


6.15 Research Questions



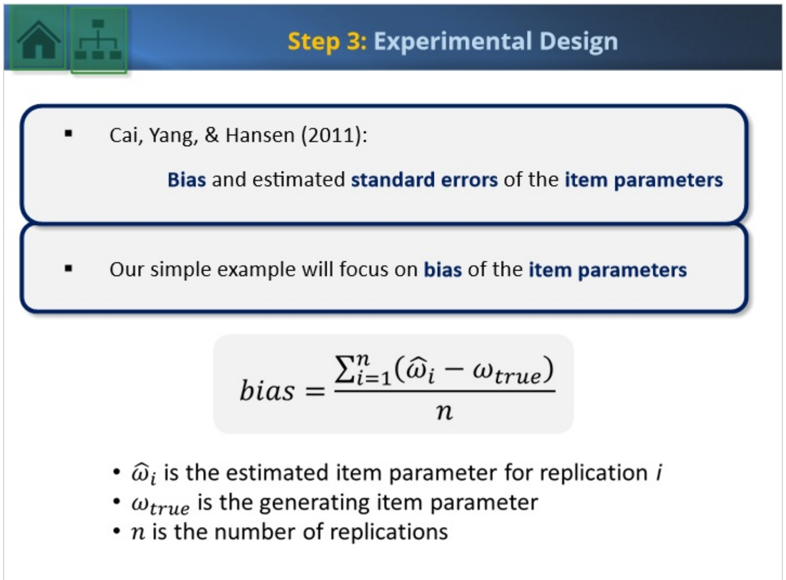
6.16 Bookend: Step 1

6.17 Justifying Conditions



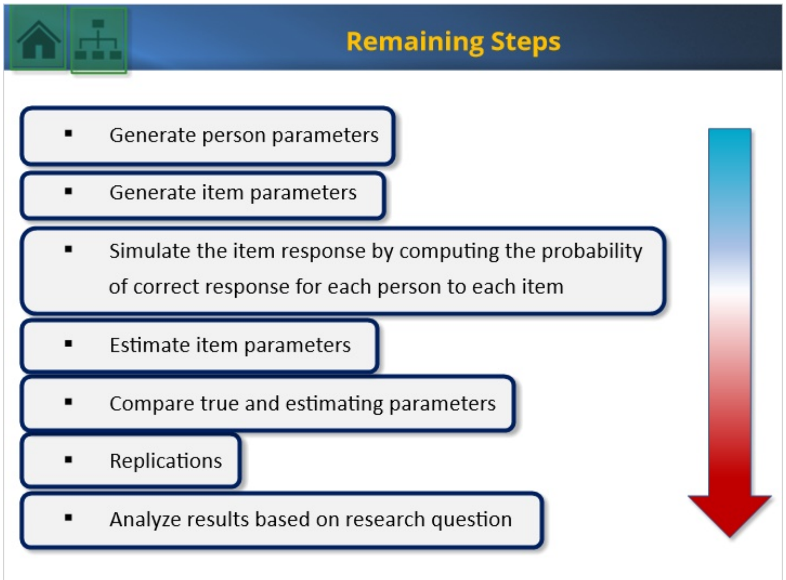
6.18 Bookend: Step 2

6.19 Experimental Design

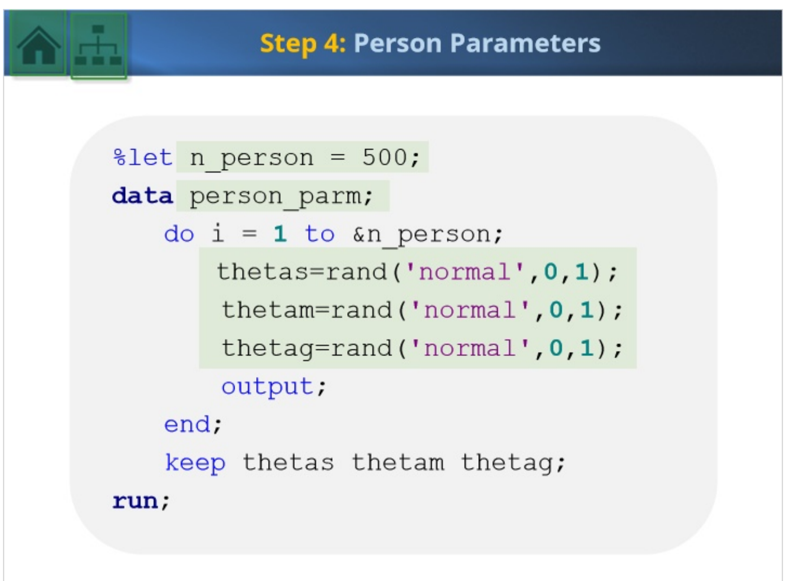


6.20 Bookend: Step 3

6.21 Remaining Steps



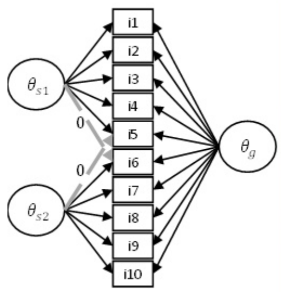
6.22 Person Parameters



6.23 Item Parameters

Step 4: Item Parameters

- Borrow item parameters from DeMars (2013)
- Loadings not in the model are fixed to 0



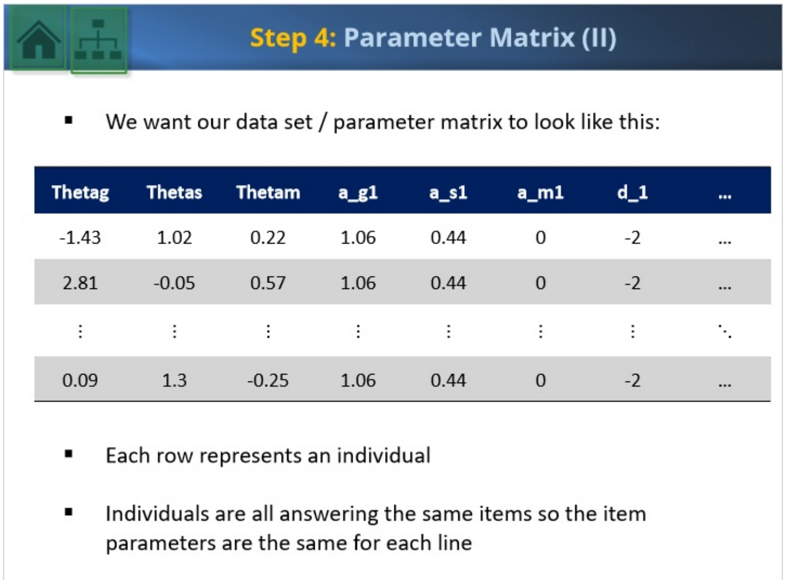
Item	a_{jg}	a_{js}	a_{jm}	d_j
1	1.06	0.44	0	-0.01
2	1.59	0.45	0	-0.13
3	0.98	0.36	0	0.44
4	0.88	0.48	0	-0.34
5	0.27	0.33	0	1.26
6	1.04	0	0.51	-0.77
7	0.61	0	0.23	-0.07
8	1.34	0	0.45	-0.14
9	0.96	0	0.46	2.22
10	1.20	0	0.42	0.18

6.24 Parameter Matrix (I)

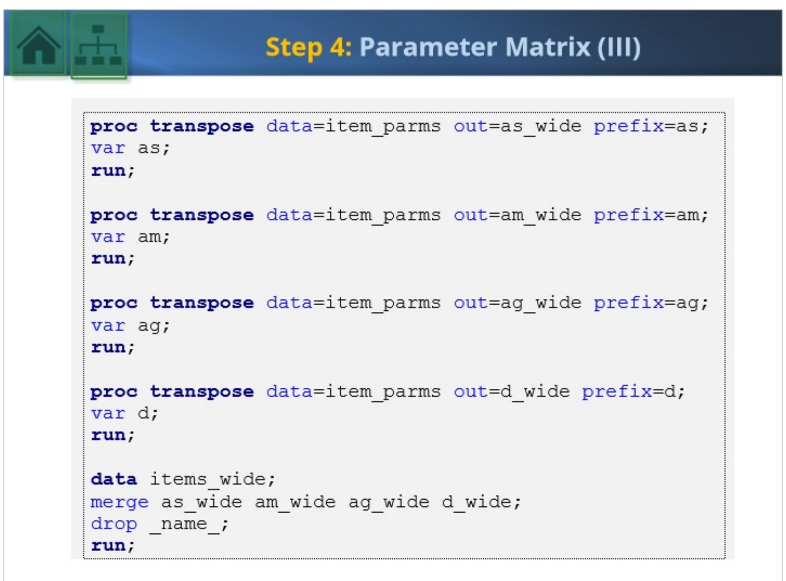
Step 4: Parameter Matrix (I)

```
data item_parms;
input ag as am d;
cards;
1.06 .44 0 -.01
1.59 .45 0 -.13
0.98 .36 0 0.44
0.88 .48 0 -.34
0.27 .33 0 1.26
1.04 0 .51 -.77
0.61 0 .23 -.07
1.34 0 .45 -.14
0.96 0 .46 2.22
1.20 0 .42 0.18
;
run;
```

6.25 Parameter Matrix (II)



6.26 Parameter Matrix (III)



6.27 Parameter Matrix (IV)

Step 4: Parameter Matrix (IV)

- Let's consider one individual and the first item:

thetas	thetam	thetag	as1	am1	ag1	d1
1.54	-1.30	-0.81	0.44	0	1.06	-0.01

- We want a data file that looks like this:

Item 1	Item 2	Item 3	...	Item 10
1	1	0	...	1
0	1	1	...	0
⋮	⋮	⋮	⋱	⋮
0	1	1	...	0

6.28 Parameter Matrix (V)

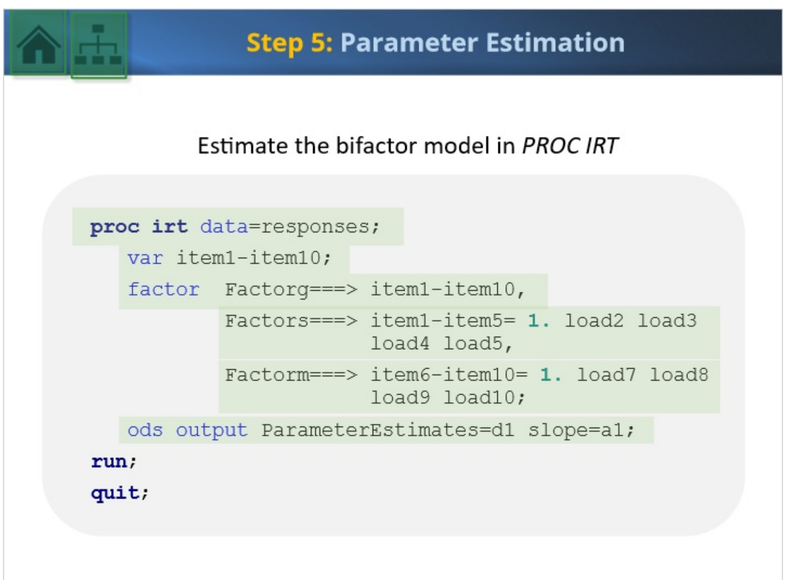
Step 4: Parameter Matrix (V)

```
data responses;
set all_parms;
   array as{&n_item} as1-as&n_item;
   array am{&n_item} am1-am&n_item;
   array ag{&n_item} ag1-ag&n_item;
   array d{&n_item} d1-d&n_item;
   array p{&n_item} p1-p&n_item;
   array item{&n_item} item1 - item&n_item;
   do j = 1 to &n_item;
      p[j]=1/(1+exp(-(as[j]*thetas+am[j]*thetam+ag[j]*thetag+d[j]))
      y=rand('uniform');
     if y<=p[j] then item[j]=1;
     else if y>p[j] then item[j]=0;
   end;
run;
```

$$P_1(U_1 = 1|\omega_j, \boldsymbol{\theta}) = \frac{1}{1 + e^{-(a_{s1}\theta_s + a_{m1}\theta_m + a_{g1}\theta_g + d_1)}}$$

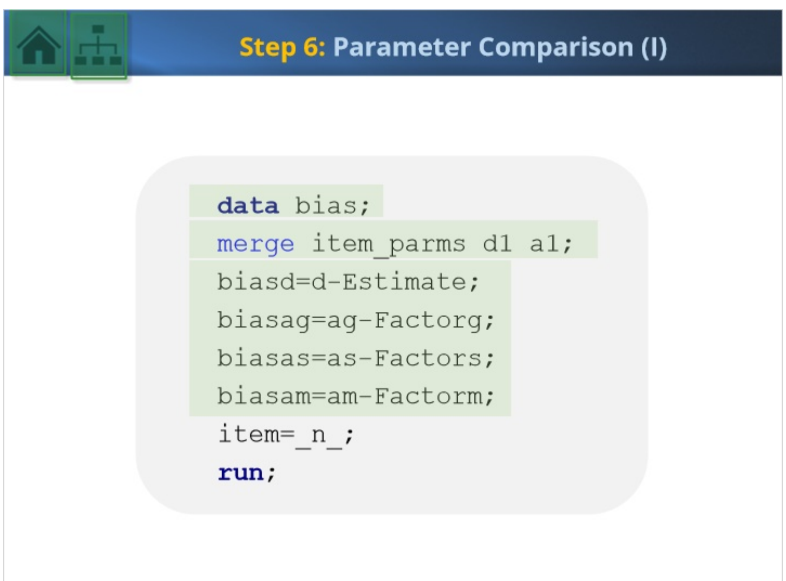
6.29 Bookend: Step 4

6.30 Parameter Estimation

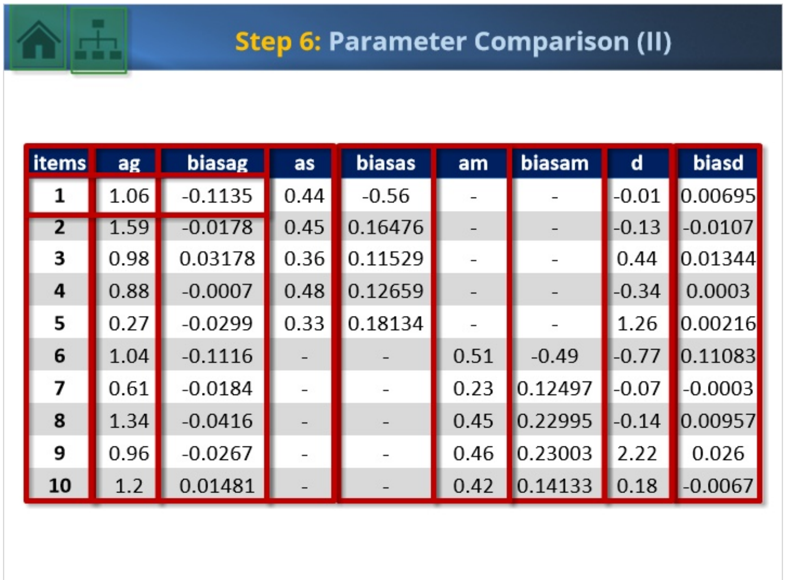


6.31 Bookend: Step 5

6.32 Parameter Comparison (I)



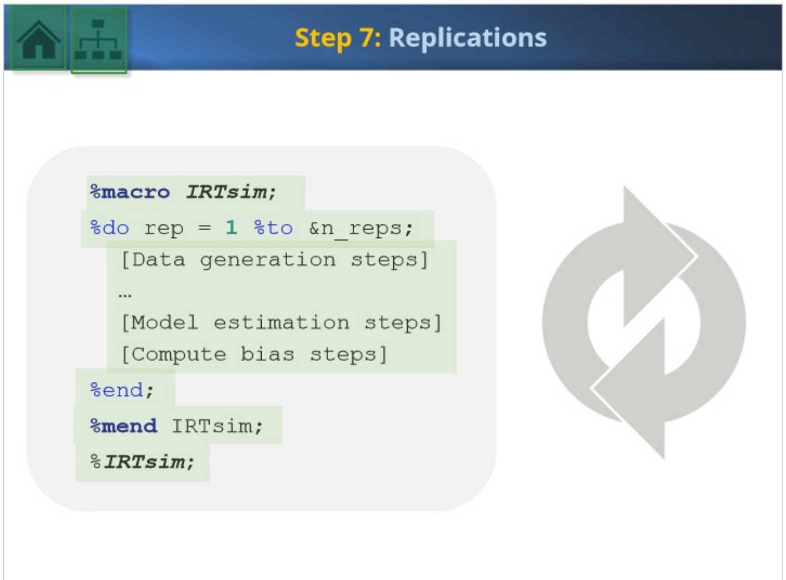
6.33 Parameter Comparison (II)



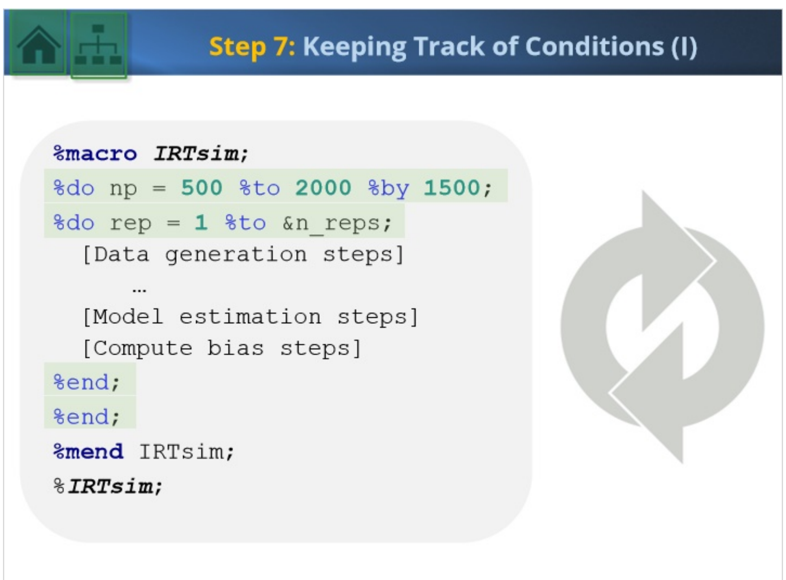
items	ag	biasag	as	biasas	am	biasam	d	biasd
1	1.06	-0.1135	0.44	-0.56	-	-	-0.01	0.00695
2	1.59	-0.0178	0.45	0.16476	-	-	-0.13	-0.0107
3	0.98	0.03178	0.36	0.11529	-	-	0.44	0.01344
4	0.88	-0.0007	0.48	0.12659	-	-	-0.34	0.0003
5	0.27	-0.0299	0.33	0.18134	-	-	1.26	0.00216
6	1.04	-0.1116	-	-	0.51	-0.49	-0.77	0.11083
7	0.61	-0.0184	-	-	0.23	0.12497	-0.07	-0.0003
8	1.34	-0.0416	-	-	0.45	0.22995	-0.14	0.00957
9	0.96	-0.0267	-	-	0.46	0.23003	2.22	0.026
10	1.2	0.01481	-	-	0.42	0.14133	0.18	-0.0067

6.34 Bookend: Step 6

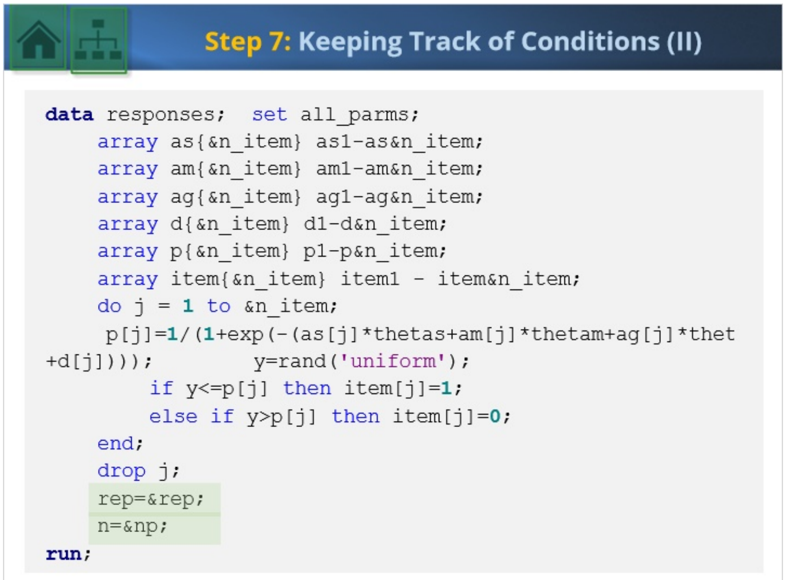
6.35 Replications (I)



6.36 Replications (II)

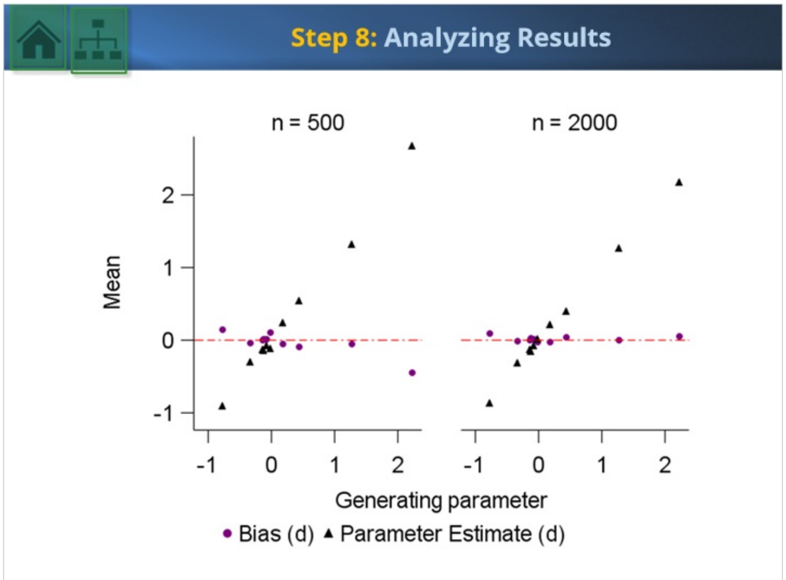


6.37 Replications (III)



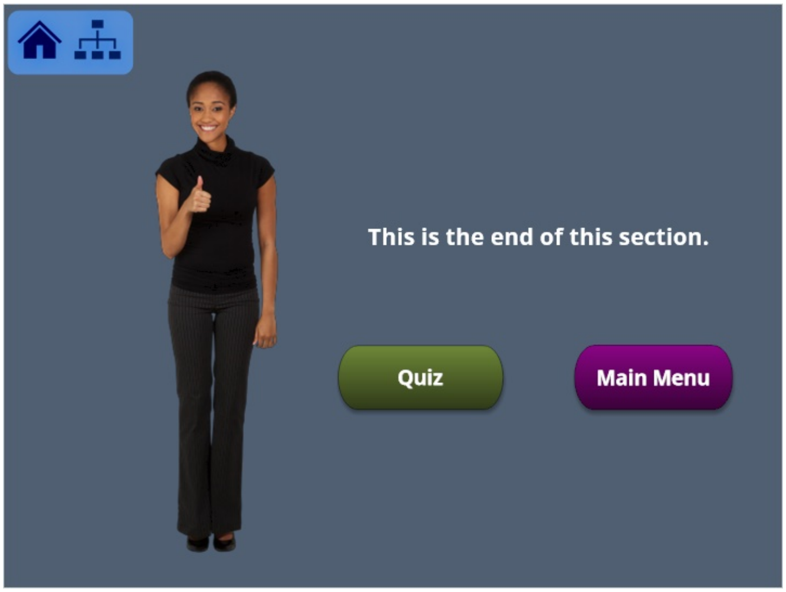
6.38 Bookend: Step 7

6.39 Analyzing Results



6.40 Bookend: Step 8

6.41 Bookend: Section 5



6.42 Module Cover (END)