

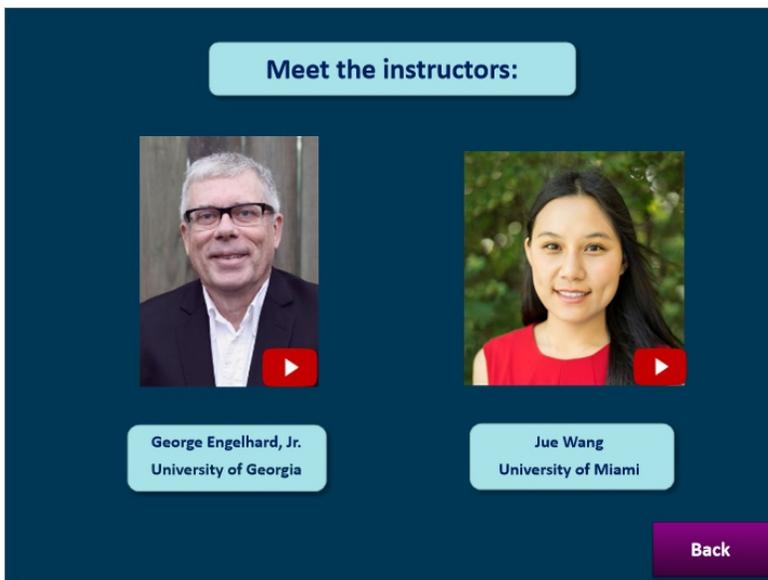
# DM10 SLIDES (Rasch Measurement Theory, Version 2.1)

## 1. Module Overview

### 1.1 Module Cover (START)



### 1.2 Instructors



### 1.3 Designers

Meet the designers:



Jonathan Lehrfeld  
ETS



André A. Rupp  
Mindful  
Measurement

Back

### 1.4 Welcome



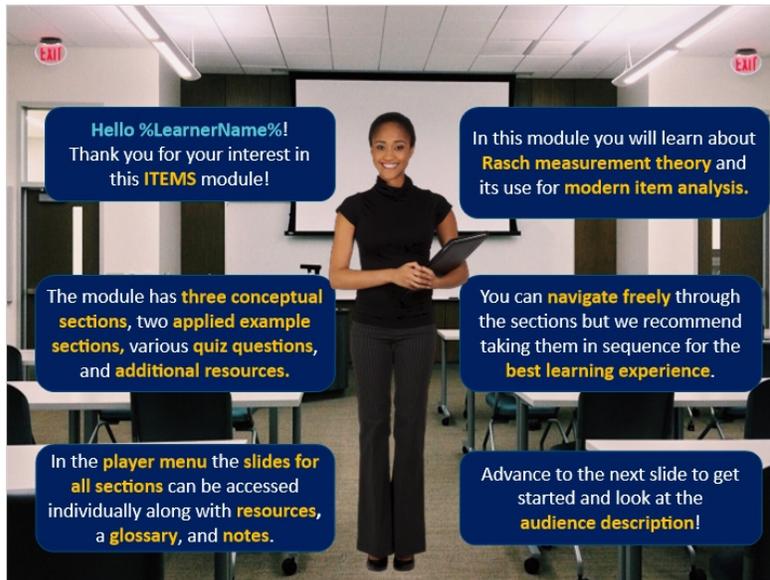
**Welcome to the  
ITEMS Module!**

The woman to the left is Laura!

Along with the instructors,  
she will be guiding you  
through the module content.

Tell us your name here:

## 1.5 Overview



## 1.6 Target Audience

**Target Audience**

Anyone who would like a gentle statistical introduction to this topic:

- graduate students and faculty in Master's, Ph.D., or certificate programs
- psychometricians and other measurement professionals
- data scientists / analysts
- research assistants or research scientists
- technical project directors
- assessment developers

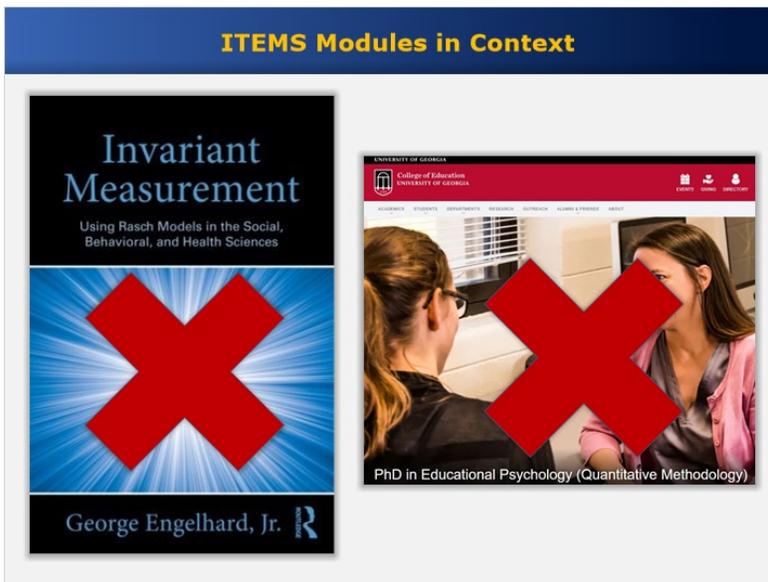


However, we hope that you find the information in this module useful no matter what your official title or role in an organization is!

## 1.7 Expectations (I)



## 1.8 Expectations (II)



## 1.9 Learning Objectives

**Learning Objectives**



1. Describe key components of Rasch measurement theory
2. Understand invariant measurement and Rasch's philosophy of measurement
3. Apply Rasch measurement theory to scale construction
4. Perform Rasch model analysis using computer programs
5. Evaluate the fit of a Rasch scale and interpret Rasch location measures
6. Make suggestions and implications to scale development in operational practice

## 1.10 Prerequisites

**Prerequisites**

**Working knowledge of foundational assessment and statistical concepts such as:**

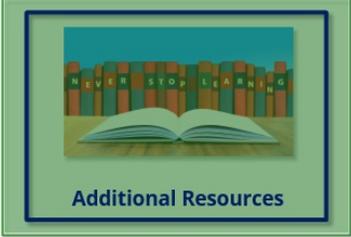
- ✓ Basic descriptive statistics (e.g., mean, standard deviation)
- ✓ Basic mathematical functions (e.g., logarithms)
- ✓ Statistical definition of probability
- ✓ Guttman and Thurstone scaling
- ✓ Other measurement theories:
  - classical test theory (CTT)
  - generalizability theory (g-theory)
  - item response theory (IRT)

## 1.11 Resources

### Resources

**Module Citation**

Wang, J., & Engelhard Jr., G., (2019). Rasch measurement theory (Digital ITEMS Module 10). *Educational Measurement: Issues and Practice*, 38(4), 112-113.



Additional Resources

## Resources 2 (Slide Layer)

### Resources: Books

Engelhard & Wind (2018). *Invariant Measurement with Raters and Rating Scales: Rasch Models for Rater-Mediated Assessments*.



Wilson (2005). *Constructing Measures: An Item Response Modeling Approach*.



[More Books](#)

[Back](#)

## Resources 1 (Slide Layer)

**Resources: Books**

Andrich & Marais (2019). *A Course In Rasch Measurement Theory*.

Bond & Fox (2015). *Applying the Rasch Model: Fundamental Measurement In the Human Sciences*.

Engelhard (2013). *Invariant Measurement: Using Rasch Models in the Social, Behavioral, and Health Sciences*.

[More Books](#) [Back](#)

### 1.12 Main Menu

**Main Menu**

**Theory**

- 01 Conceptual Foundations [20 Minutes]
- 02 Rasch Measurement Theory [30 Minutes]
- 03 Creating a Rasch Scale [25 Minutes]

**Practice**

- 04 Winsteps and Shiny ERMA [20 Minutes]
- 05 Data Activities [30 Minutes]
- 06 Quizzes [30 Minutes]

## 2. Section 1: Conceptual Foundations

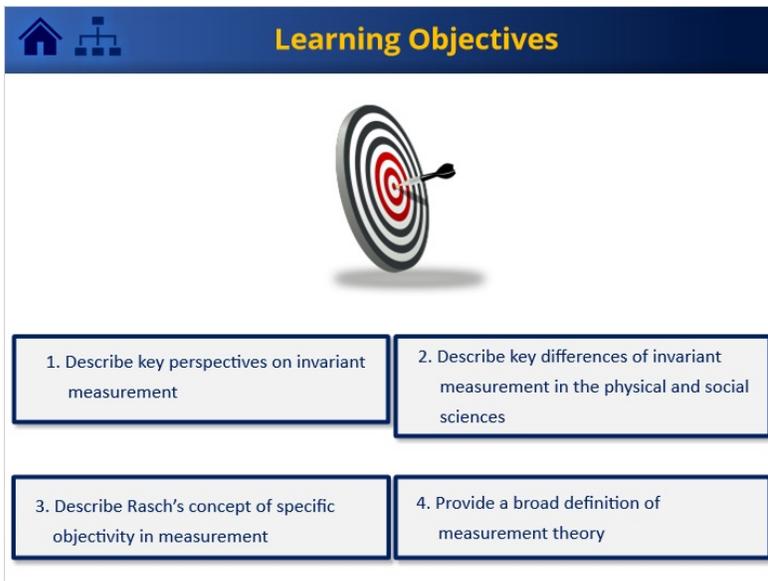
### 2.1 Cover: Section 1



The cover slide features a photograph of a classroom with a projector screen displaying "Hello %LearnerName%!". A dark blue sidebar on the right contains the text "Section 1: Conceptual Foundations (20 Minutes)". A navigation bar at the top left includes a home icon and a tree icon.

Section 1:  
Conceptual  
Foundations  
(20 Minutes)

### 2.2 Objectives: Section 1

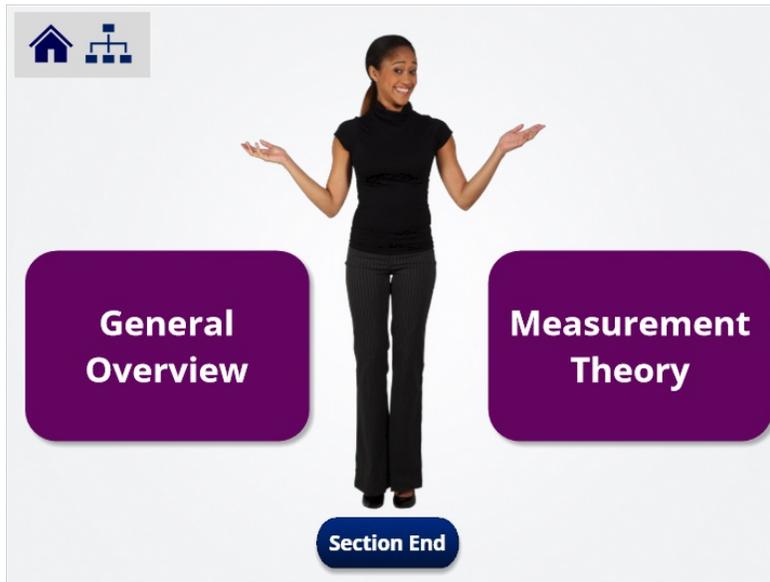


The slide is titled "Learning Objectives" and features a target icon with an arrow in the bullseye. Below the icon are four numbered objectives in separate boxes. A navigation bar at the top left includes a home icon and a tree icon.

Learning Objectives

1. Describe key perspectives on invariant measurement
2. Describe key differences of invariant measurement in the physical and social sciences
3. Describe Rasch's concept of specific objectivity in measurement
4. Provide a broad definition of measurement theory

## 2.3 Topic Selection



## 2.4 Bookmark: Overview



## 2.5 Perspectives on Invariant Measurement



### General Overview

Invariant Measurement

Objectivity

Network of Measures



Click on the buttons to see quotes from seminal researchers 

### Invariant Measurement (Slide Layer)



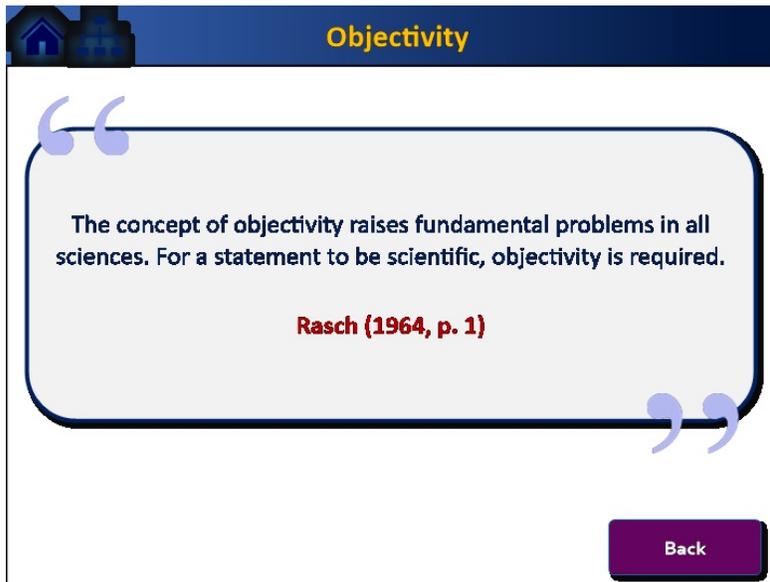
### Invariant Measurement

“  
The scientist is usually looking for invariance whether he knows it or not... The scientist seeks measures that will stay put while his back is turned.  
”

**Stevens (1951, pp. 20-21)**

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## Objectivity (Slide Layer)



The slide features a dark blue header with a home icon and the title "Objectivity" in yellow. The main content is a light gray rounded rectangle with a dark blue border, containing a quote in black text and the citation "Rasch (1964, p. 1)" in red. The quote is enclosed in large, light blue quotation marks. A purple "Back" button is located in the bottom right corner.

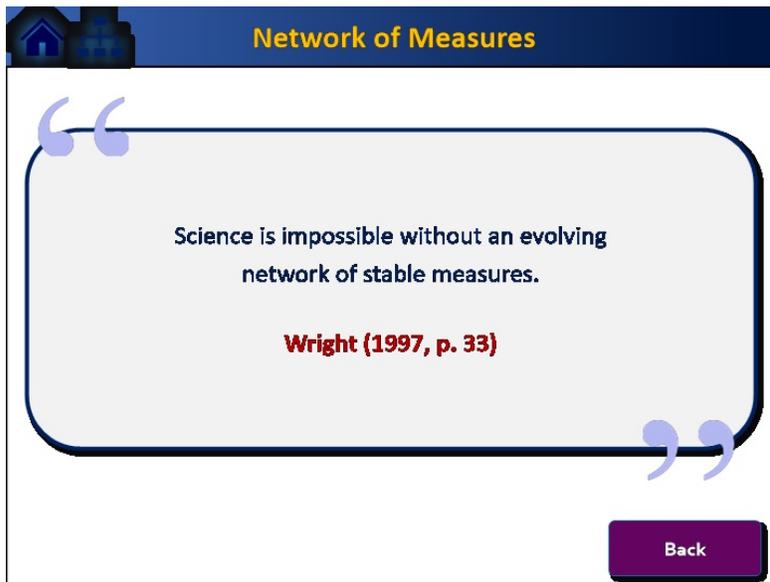
**Objectivity**

The concept of objectivity raises fundamental problems in all sciences. For a statement to be scientific, objectivity is required.

**Rasch (1964, p. 1)**

Back

## Network of Measures (Slide Layer)



The slide features a dark blue header with a home icon and the title "Network of Measures" in yellow. The main content is a light gray rounded rectangle with a dark blue border, containing a quote in black text and the citation "Wright (1997, p. 33)" in red. The quote is enclosed in large, light blue quotation marks. A purple "Back" button is located in the bottom right corner.

**Network of Measures**

Science is impossible without an evolving network of stable measures.

**Wright (1997, p. 33)**

Back

## 2.6 Invariant Measurement in Physical Sciences

  **Invariant Measurement: Physical Sciences**

<p><b>Measurement of length</b></p> 	<p><b>Measurement of weight</b></p> 
<p><b>Measurement of time</b></p> 	<p><b>Measurement of temperature</b></p> 

## 2.7 Invariant Measurement in Social Sciences 1

  **Invariant Measurement: Social Sciences**

**Wong-Baker FACES® Pain Rating Scale**

					
<b>0</b>	<b>2</b>	<b>4</b>	<b>6</b>	<b>8</b>	<b>10</b>
No Hurt	Hurts Little Bit	Hurts Little More	Hurts Even More	Hurts Whole Lot	Hurts Worst

- developed by Donna Wong and Connie Baker
- originally created for children, but now used with patients age 3 and above across the world

(Source: <https://wongbakerfaces.org/>)

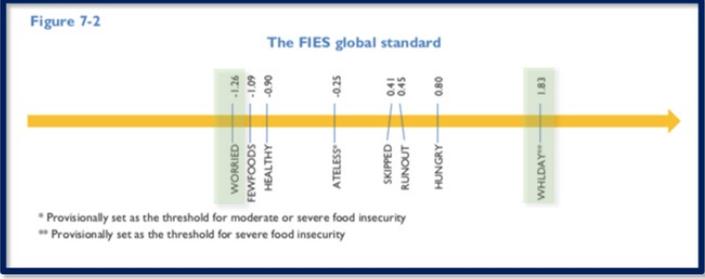
**WONG-BAKER FACES FOUNDATION**

## 2.8 Invariant Measurement in Social Sciences 2

  **Invariant Measurement: Social Sciences**

**Figure 7-2**

The FIES global standard



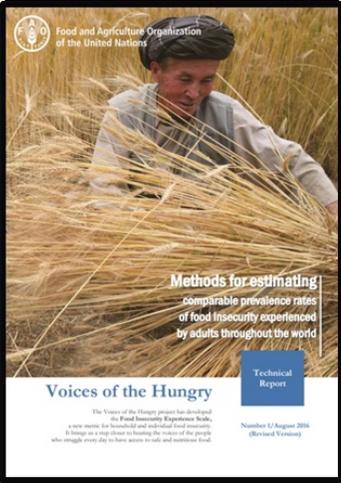
\* Provisionally set as the threshold for moderate or severe food insecurity  
\*\* Provisionally set as the threshold for severe food insecurity

A global scale for measuring food insecurity experience among adults

[Reference](#)

### References (Slide Layer)

  **Reference**



Food and Agriculture Organization of the United Nations (UN-FAO). (2016). *Voices of the hungry: Methods for estimating comparable rates of food insecurity experienced by adults throughout the world*. Rome, FAO.

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## 2.9 Measurement in Physical vs. Social Sciences



### Physical vs. Social Sciences

The measurement of psychological constructs can be more difficult than the measurement of physical objects since constructs cannot be directly observed

**Two difficulties for scales:**

- Different instruments designed to measure the same construct may provide different measures for the same person
- The same instrument may work in different ways for different persons



## 2.10 Specific Objectivity of Measurement 1



### Invariant Measurement

Thurstone → Guttman → Rasch

Similarities	Differences
<p>Proposed the concept of invariance (specific objective) of comparison.</p>	<p>Thurstone focused on groups, while Rasch developed individual-centered statistics.</p>
<p>Emphasized the demand for unidimensionality for achieving invariance.</p>	<p>Guttman's scale is deterministic, while Rasch formalized a probabilistic model with both items and persons parameterized.</p>

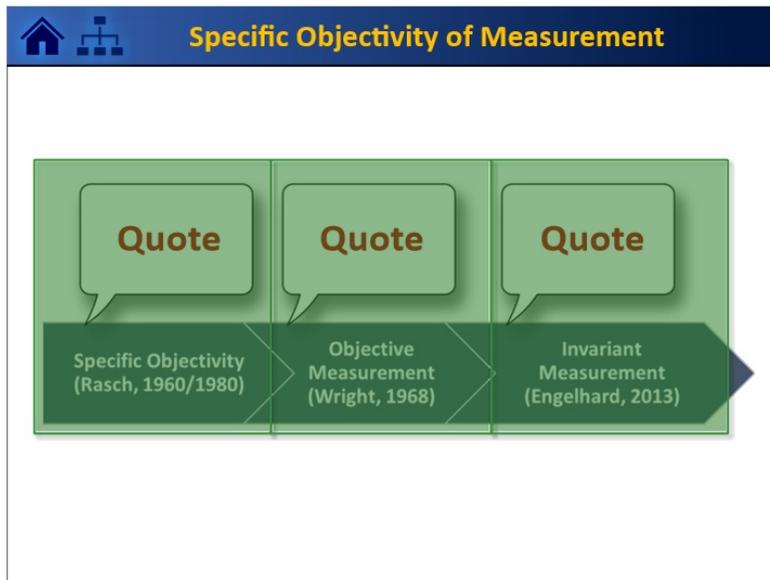
[References](#)

## References (Slide Layer)



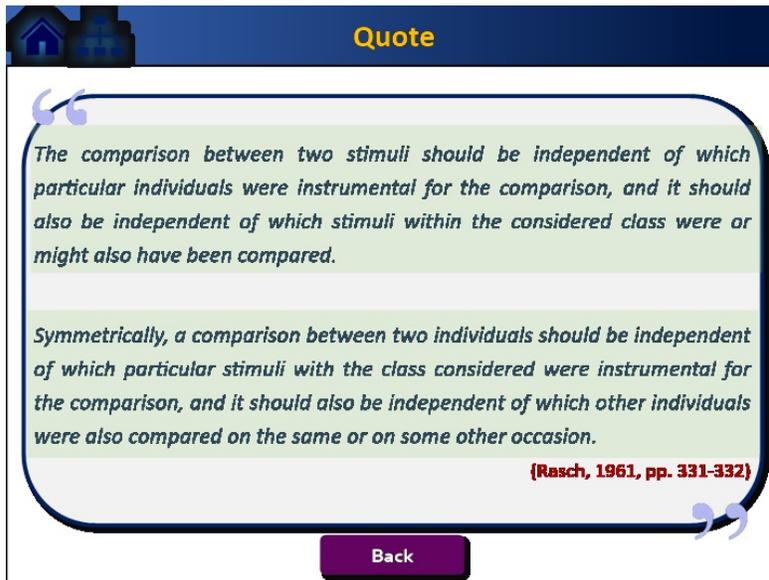
The slide features a dark blue header with a home icon and the title "References" in yellow. Below the header are two large, empty green rectangular boxes. A purple "Back" button is located in the bottom right corner.

## 2.11 Specific Objectivity of Measurement 2



The slide features a dark blue header with a home icon, a tree icon, and the title "Specific Objectivity of Measurement" in yellow. Below the header is a diagram with three green boxes. Each box contains a "Quote" in a speech bubble above a measurement concept: "Specific Objectivity (Rasch, 1960/1980)", "Objective Measurement (Wright, 1968)", and "Invariant Measurement (Engelhard, 2013)". Arrows point from left to right between the boxes.

## Rasch Quote (Slide Layer)



**Quote**

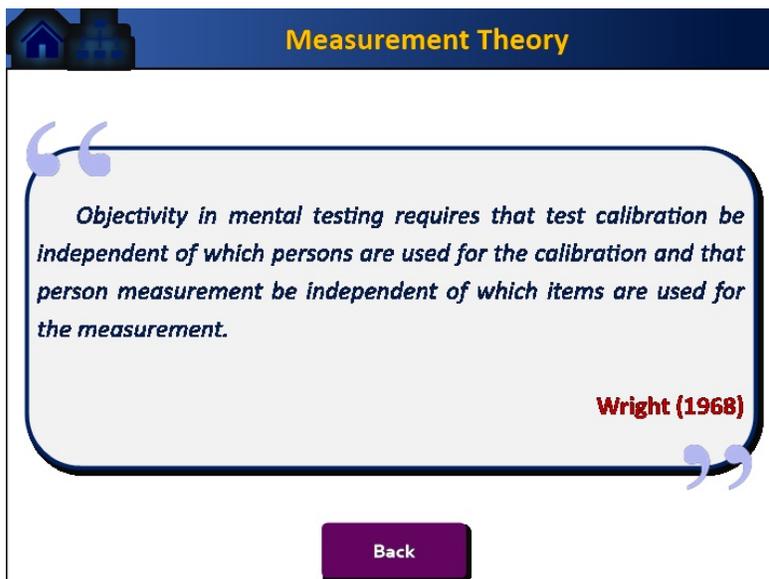
*The comparison between two stimuli should be independent of which particular individuals were instrumental for the comparison, and it should also be independent of which stimuli within the considered class were or might also have been compared.*

*Symmetrically, a comparison between two individuals should be independent of which particular stimuli with the class considered were instrumental for the comparison, and it should also be independent of which other individuals were also compared on the same or on some other occasion.*

**(Rasch, 1961, pp. 331-332)**

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## Wright Quote (Slide Layer)



**Measurement Theory**

*Objectivity in mental testing requires that test calibration be independent of which persons are used for the calibration and that person measurement be independent of which items are used for the measurement.*

**Wright (1968)**

Back

## Engelhard Quote (Slide Layer)

### Five Requirements for Invariant Measurement

#### Person measurement:

1. The measurement of persons must be independent of the particular items that happen to be used for the measuring: item-invariant measurement of persons.
2. A more able person must always have a better chance of success on an item than a less able person: non-crossing person response functions.

#### Item calibration:

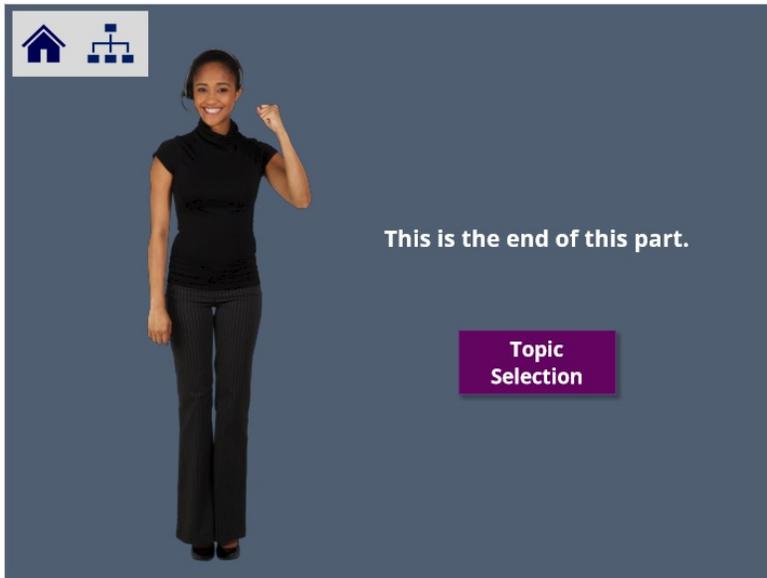
3. The calibration of the items must be independent of the particular persons used for calibration: person-invariant calibration of test items.
4. Any person must have a better chance of success on an easy item than on a more difficult item: non-crossing item response functions.

#### Variable map:

5. Items and persons must be simultaneously located on a single underlying latent variable: variable map.

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## 2.12 Bookend: Overview



The slide features a woman in a black top and pants on the left. In the top left corner, there are two icons: a house and a tree diagram. The text 'This is the end of this part.' is centered on the right. A purple button labeled 'Topic Selection' is positioned below the text.

## 2.13 Bookmark: Measurement Theory



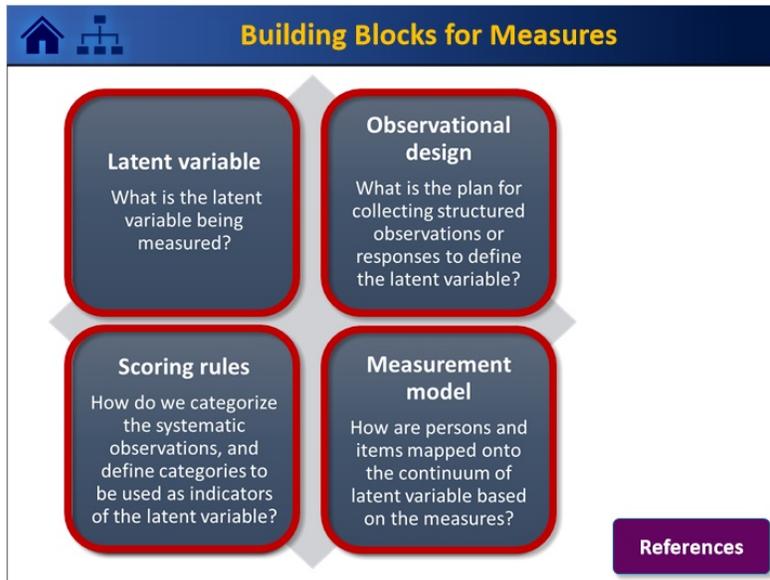
## 2.14 Measurement Theory

Measurement Theory

“  
Theories of measurement broadly conceived may be viewed as loosely integrated **conceptual frameworks** within which are embedded rigorously formulated **statistical models** of estimation and inference about the **properties of measurements and scores**.  
”

Messick (1983, p. 498)

## 2.15 Building Blocks for Researcher-Constructed Measures



**Building Blocks for Measures**

**Latent variable**  
What is the latent variable being measured?

**Observational design**  
What is the plan for collecting structured observations or responses to define the latent variable?

**Scoring rules**  
How do we categorize the systematic observations, and define categories to be used as indicators of the latent variable?

**Measurement model**  
How are persons and items mapped onto the continuum of latent variable based on the measures?

**References**

### References (Slide Layer)



**References**

**Invariant Measurement**  
Using Rasch Models in the Social, Behavioral, and Health Sciences  
George Engelhard, Jr. R

**Constructing Measures**  
An Item Response Modeling Approach  
Mark Wilson

**Back**

## 2.16 Measurement/Statistical Models

Test-Score Tradition	Scaling Tradition
<p><b>Key models:</b></p> <ol style="list-style-type: none"><li>1. Classical test theory (CTT)</li><li>2. Generalizability theory (GT)</li><li>3. Factor analysis (FA)</li></ol>	<p><b>Key models:</b></p> <ol style="list-style-type: none"><li>1. Psychophysical models (PM)</li><li>2. Absolute Scaling (AS)</li><li>3. Item Response Theory (IRT)</li></ol>
<p><b>Essential features:</b></p> <p>Test-score focus Linear models Focus on test scores and the estimation of error components</p>	<p><b>Essential features:</b></p> <p>Item-person response focus Non-linear models Focus on modeling the responses of persons to items</p>

## 2.17 Properties of Measurements and Scores

**The meanings and interpretations of scores are essential**

- Determine impact of research in the social, behavioral, and health sciences
- Frame the substantive conclusions and inferences that we draw
- Delineate and limit policies and practices derived from research work

*Validity is not a property of the test or assessment as such, but rather of the meaning of the test scores. These scores are a function not only of the items or stimulus conditions, but also of the persons responding as well as the context of the assessment...*

**Messick (1995, p. 741)**

## 2.18 Scores on Educational Assessments



### Scores on Educational Assessments

**The key purpose of educational assessments is to make decisions on the basis of scores**

- Levels (international, national, state, school, teacher, or student)
- Operational definitions (of the outcomes that we value as a society)

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**Classroom assessments focus on individual students**

- Before instruction: readiness, placement
- During instruction: formative, diagnostic
- After instruction: summative, strategic

[Quote](#)

### Quote (Slide Layer)



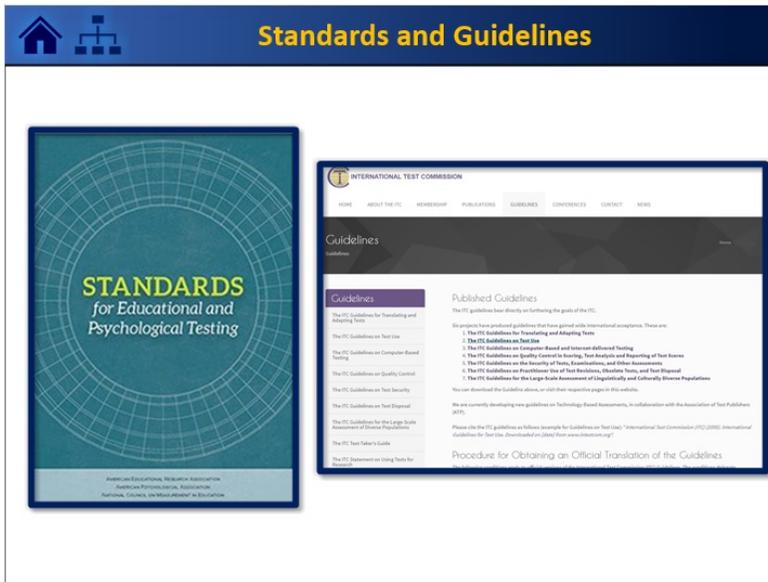
### Quote

*... the term score is used generically in its broadest sense to mean any coding or summarization of observed consistencies or performance regularities on a test, questionnaire, observation procedure, or other assessment devices such as work samples, portfolios, and realistic problem simulations.*

**Messick (1995, p. 741)**

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## 2.19 Standards & Guidelines



The screenshot displays the International Test Commission (ITC) website. On the left is the cover of the 'STANDARDS for Educational and Psychological Testing' book, published by the American Educational Research Association, Educational Psychological Association, and the International Association of Applied Psychology. On the right is a screenshot of the ITC website's 'Guidelines' page, which lists various published guidelines such as 'The ITC Guidelines for Test Use', 'The ITC Guidelines on Computer-Based Testing', and 'The ITC Guidelines on Quality Control'. The website header includes navigation links for Home, About the ITC, Membership, Publications, Guidelines, Conferences, Contact, and News.

## 2.20 Bookend: Measurement Theory



The slide features a woman in a black top and pants standing on the left. To her right, the text reads 'This is the end of this part.' Below this text is a purple button with the text 'Topic Selection'. In the top left corner, there are icons for a home page and a navigation menu.

## 2.21 Summary I



### Summary (I)

- Invariant measurement is crucial for **developing measures** for the social sciences, but it is hard to achieve since **constructs are latent**
- Rasch's concept of **specific objectivity** emphasizes the **invariance of comparisons** between stimuli and individuals
- His **theory of measurement** includes the formulation of a **measurement model** based on a conceptual framework
- The scores from this model can be used to make **inferences and predictions** about human characteristics and behaviors

Quote

### Quote (Slide Layer)



### Quote

*Looking then for concepts [of measurement] that could possibly be taken as primary it seems worth-while to concentrate upon two essential characteristics of scientific statements: 1. they are concerned with comparisons; 2. the statements are claimed to be objective; both terms of course calling for precise qualifications.*

**Rasch (1964, p. 2)**

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## 2.22 Summary II



### Summary (II)

**Item-invariant measurement of persons:**

- Measurement of persons shall be free of test forms being used
- Relevant measurement problems include test equating, item parameter linking, and computerized adaptive testing

---

**Person-invariant calibration of items:**

- Calibration of items shall be independent of specific samples
- Relevant measurement problem include differential item functioning and measurement invariance

## 2.23 Bookend: Section 1



This is the end of this section.

[Quiz](#) [Main Menu](#)

## 3. Section 2: Rasch Measurement Theory

### 3.1 Cover: Section 2



Section 2

Rasch Measurement Theory

(30 Minutes)

### 3.2 Objectives: Section 2



Learning Objectives



1. Explain Rasch's motivation for his measurement theory
2. Describe the fundamental principles of Rasch measurement theory
3. List the Rasch-based fit indices and explain their underlying structure
4. List commonly used Rasch models and explain their structure

### 3.3 Rasch's Motivation



## Rasch's Motivation



Georg Rasch, 1901-1980

Quote

#### Quote (Slide Layer)



## Rasch's Motivation

Lately the statistical methods of psychometrics have been severely criticized in psychological quarters. Thus Skinner maintains that if order is to be found in human and animal behavior, then it should be extracted from investigations into individuals, and that psychometric methods are inadequate for such purposes since they deal with groups of individuals. And as regards abnormal psychology Zubin states: "Recourse must be had to individual statistics, treating each patient as a separate universe. Unfortunately, present day statistical methods are entirely group-centered, so that there is a real need for developing individual-centered statistics."

Rasch, G. (1961). On general laws and meaning of measurement in psychology. In J. Neyman (Ed.), *Proceedings of the fourth Berkeley Symposium on mathematical statistics and probability* (pp 321-333). Berkeley: University of California Press.

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### 3.4 Rasch's Example I

Rasch's Example: Setup							
Person ID	Item ID						# Correct (Person) / Raw Score
	1	2	3	4	5	6	
1	+	+	o	+	o	+	4
2	o	o	o	+	o	o	1
3	+	+	+	+	+	+	6
4	o	+	o	+	o	o	2
5	+	+	+	+	+	+	6
6	+	+	o	+	o	+	4
7	+	+	o	+	+	+	5
8	o	o	o	o	o	o	0
9	+	+	o	+	+	+	5
10	o	+	o	+	o	o	2
# Correct (Item) / Difficulty	6	8	2	9	4	6	

A fictitious example of **10 persons** solving **6 items** (+: correct; o: incorrect)

### 3.5 Rasch's Example II

Rasch's Example: Item Ordering							
Person ID	Item ID						# Correct (Person) / Raw Score
	4	2	1	6	5	3	
1	+	+	+	+	o	o	4
2	+	o	o	o	o	o	1
3	+	+	+	+	+	+	6
4	+	+	o	o	o	o	2
5	+	+	+	+	+	+	6
6	+	+	+	+	o	o	4
7	+	+	+	+	+	o	5
8	o	o	o	o	o	o	0
9	+	+	+	+	+	o	5
10	+	+	o	o	o	o	2
# Correct (Item) / Difficulty	9	8	6	6	4	2	

Order items according to difficulty by the **number of correct answers** to each item

### 3.6 Rasch's Example III

**Rasch's Example: Person Ordering**

Person ID	Item ID						# Correct (Person) / Raw Score
	4	2	1	6	5	3	
3	+	+	+	+	+	+	6
5	+	+	+	+	+	+	6
7	+	+	+	+	+	o	5
9	+	+	+	+	+	o	5
1	+	+	+	+	o	o	4
6	+	+	+	+	o	o	4
4	+	+	o	o	o	o	2
10	+	+	o	o	o	o	2
2	+	o	o	o	o	o	1
8	o	o	o	o	o	o	0
# Correct (Item) / Difficulty	9	8	6	6	4	2	

Order persons by their "ability" to solve each single item by using their raw scores

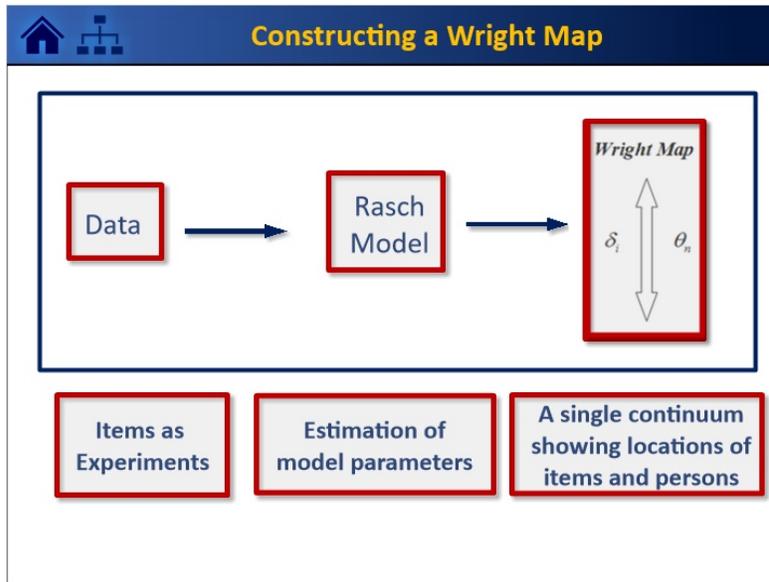
### 3.7 Guttman Scale

**Deterministic Guttman Scales**

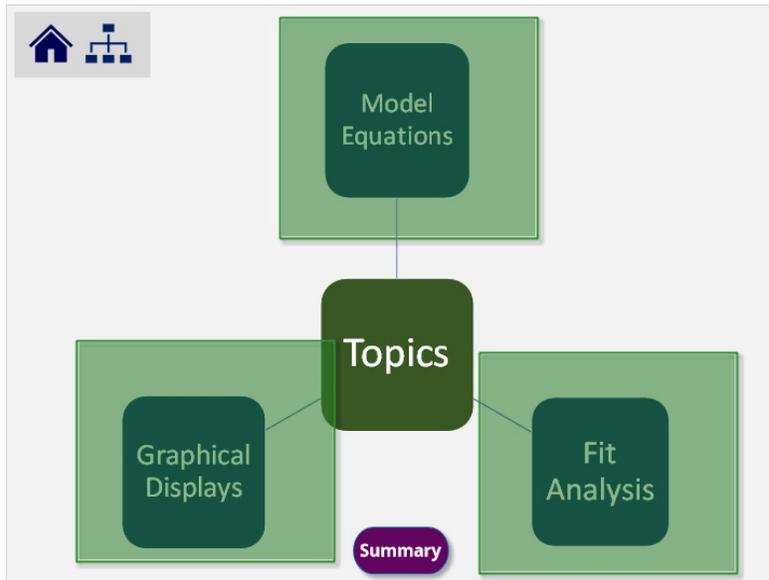
- Designed to create a **single continuum** for the **joint ordering** of persons and items
- Response patterns are **completely determined**
- Sets the foundation for item analysis that is closely related to **probabilistic scaling**
- We want to produce a probabilistic **Guttman response pattern**
- The reproduced **Rasch-based probabilities** can be used to calculate **residuals** and **fit indices**



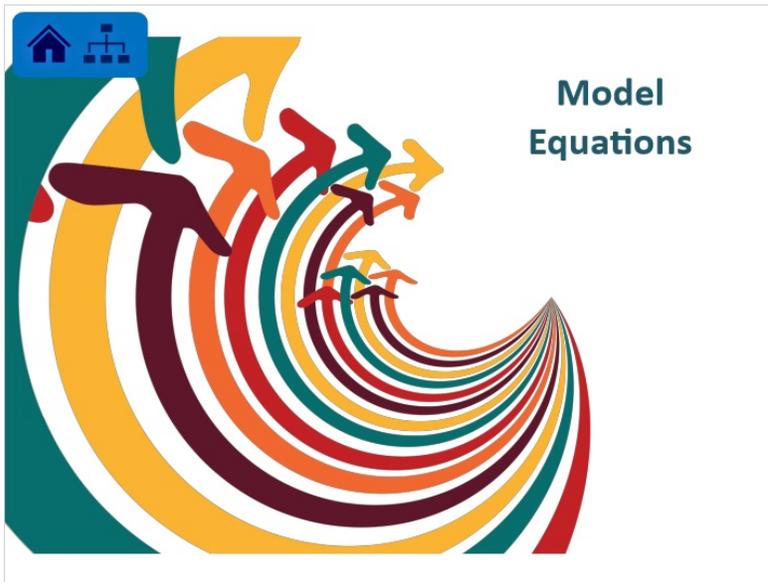
### 3.8 Constructing a Wright Map



### 3.9 Topic Selection



### 3.10 Bookmark: Equations



### 3.11 Original Form of Rasch Model

**Original Form of Rasch Model**

- Probability of solving an item is determined by the degree of **ability of the person** ( $\xi_n$ ) and the degree of **difficulty of the item** ( $\delta_i$ )
- Probability is a **joint function** of  $\xi_n/\delta_i$  instead of  $\xi_n$  and  $\delta_i$  separately
- The simplest **probability function** is:

$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

Hover over parameters to see explanation 

## ksi (Slide Layer)



### Original Form of Rasch Model

- Probability of solving an item is determined by the degree of **ability of the person** ( $\xi_n$ ) and the degree of **difficulty of the item** ( $\delta_i$ )
- Probability is a **joint function** of  $\xi_n/\delta_i$  instead of  $\xi_n$  and  $\delta_i$  separately
- The simplest **probability function** is:

$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

Ability of person  $n$

Hover over parameters to see explanation 

## delta (Slide Layer)



### Original Form of Rasch Model

- Probability of solving an item is determined by the degree of **ability of the person** ( $\xi_n$ ) and the degree of **difficulty of the item** ( $\delta_i$ )
- Probability is a **joint function** of  $\xi_n/\delta_i$  instead of  $\xi_n$  and  $\delta_i$  separately
- The simplest **probability function** is:

$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

Difficulty of item  $i$

Hover over parameters to see explanation 

## Pni1 (Slide Layer)



### Original Form of Rasch Model

- Probability of solving an item is determined by the degree of **ability of the person** ( $\xi_n$ ) and the degree of **difficulty of the item** ( $\delta_i$ )
- Probability is a **joint function** of  $\xi_n/\delta_i$ , instead of  $\xi_n$  and  $\delta_i$  separately
- The simplest **probability function** is:

$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

Probability of person  $n$  answering item  $i$  correctly

Hover over parameters to see explanation 

### 3.12 Original Form of Rasch Model



### Sample Probabilities

- A **very able** person solving a **very easy** item  $\rightarrow 1$
- A **least able** person solving a **very difficult** item  $\rightarrow 0$
- An **average ability** person solving an **average difficulty** item ( $\xi_n = \delta_i$ ) = .50

$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

Hover over parameters to see explanation 

## ksi (Slide Layer)



### Sample Probabilities

- A **very able** person solving a **very easy** item  $\rightarrow 1$
- A **least able** person solving a **very difficult** item  $\rightarrow 0$
- An **average ability** person solving an **average difficulty** item  $(\xi_n = \delta_i) = .50$

$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

Ability of person  $n$

Hover over parameters to see explanation 

## delta (Slide Layer)



### Sample Probabilities

- A **very able** person solving a **very easy** item  $\rightarrow 1$
- A **least able** person solving a **very difficult** item  $\rightarrow 0$
- An **average ability** person solving an **average difficulty** item  $(\xi_n = \delta_i) = .50$

$$P_{ni1} = \frac{\frac{\xi_n}{\delta_i}}{1 + \frac{\xi_n}{\delta_i}} = \frac{\xi_n}{\xi_n + \delta_i}$$

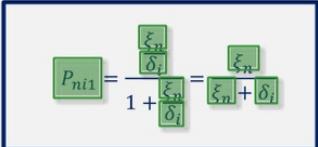
Difficulty of item  $i$

Hover over parameters to see explanation 

## Pni1 (Slide Layer)

🏠
Sample Probabilities

- A **very able** person solving a **very easy** item → 1
- A **least able** person solving a **very difficult** item → 0
- An **average ability** person solving an **average difficulty** item ( $\xi_n = \delta_i$ ) = .50



Probability of person  $n$  answering item  $i$  correctly

Hover over parameters to see explanation 

## 3.13 Dichotomous Rasch Model I

🏠
Rasch Model (I)

Rasch's Original Formulas

- Correct response:  $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response:  $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio:  $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits:  $\text{Ln} \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \text{Ln} \left[ \frac{\xi_n}{\delta_i} \right] = \text{Ln}(\xi_n) - \text{Ln}(\delta_i)$

Modern Formula

- Log-odds / Logits:  $\text{Ln} \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Hover over parameters to see explanation 

## ksi-1 (Slide Layer)

 
Rasch Model (I)

**Rasch's Original Formulas**

- Correct response:  $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$  Ability of person  $n$
- Incorrect response:  $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio:  $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[ \frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

**Modern Formula**

- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$  Hover over parameters to see explanation 

## delta-1 (Slide Layer)

 
Rasch Model (I)

**Rasch's Original Formulas**

- Correct response:  $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$  Difficulty of item  $i$
- Incorrect response:  $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio:  $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[ \frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

**Modern Formula**

- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$  Hover over parameters to see explanation 

## Pni1-1 (Slide Layer)

Rasch Model (I)

**Rasch's Original Formulas**

- Correct response:  $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response:  $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio:  $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[ \frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Probability of person  $n$  answering item  $i$  correctly

**Modern Formula**

- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Hover over parameters to see explanation

## delta-2 (Slide Layer)

Rasch Model (I)

**Rasch's Original Formulas**

- Correct response:  $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response:  $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio:  $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[ \frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

Difficulty of item  $i$

**Modern Formula**

- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Hover over parameters to see explanation

## ksi-2 (Slide Layer)

🏠
Rasch Model (I)

**Rasch's Original Formulas**

- Correct response:  $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response:  $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$  Ability of person  $n$
- Odds ratio:  $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[ \frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

**Modern Formula**

- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$  Hover over parameters to see explanation

## Pni0-2 (Slide Layer)

🏠
Rasch Model (I)

**Rasch's Original Formulas**

- Correct response:  $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response:  $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$  Probability of person  $n$  answering item  $i$  incorrectly
- Odds ratio:  $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[ \frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

**Modern Formula**

- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$  Hover over parameters to see explanation

### odds-ratio-3 (Slide Layer)

Rasch Model (I)

**Rasch's Original Formulas**

- Correct response:  $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response:  $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio:  $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$  Odds ratio of person  $n$  answering item  $i$  correctly
- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[ \frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

**Modern Formula**

- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$  Hover over parameters to see explanation

### log-odds-4 (Slide Layer)

Rasch Model (I)

**Rasch's Original Formulas**

- Correct response:  $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response:  $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio:  $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[ \frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

**Modern Formula**

- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$  Log-odds of correct response to item  $i$

Hover over parameters to see explanation

## log-odds-5 (Slide Layer)

Rasch Model (I)

**Rasch's Original Formulas**

- Correct response:  $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response:  $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio:  $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[ \frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

**Modern Formula**

- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Log-odds of correct response to item *i*

to see explanation

## theta-5 (Slide Layer)

Rasch Model (I)

**Rasch's Original Formulas**

- Correct response:  $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response:  $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio:  $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[ \frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

**Modern Formula**

- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Ability of person *n*

## delta-5 (Slide Layer)

🏠
Rasch Model (I)

**Rasch's Original Formulas**

- Correct response:  $P_{ni1} = \frac{\xi_n}{\xi_n + \delta_i}$
- Incorrect response:  $P_{ni0} = 1 - \frac{\xi_n}{\xi_n + \delta_i} = \frac{\delta_i}{\xi_n + \delta_i}$
- Odds ratio:  $\frac{P_{ni1}}{P_{ni0}} = \frac{\xi_n}{\delta_i}$
- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \ln \left[ \frac{\xi_n}{\delta_i} \right] = \ln(\xi_n) - \ln(\delta_i)$

**Modern Formula**

- Log-odds / Logits:  $\ln \left[ \frac{P_{ni1}}{P_{ni0}} \right] = \theta_n - \delta_i$

Difficulty of item *i*

to see explanation

## 3.14 Dichotomous Rasch Model II

🏠
Rasch Model (II)

**Probability Function:**

$$P_{nix} = \frac{\exp[x_{ni}(\theta_n - \delta_i)]}{\sum_{x_{ni}} \exp[x_{ni}(\theta_n - \delta_i)]}$$

**Response Probability:**

Correctly Answer

Incorrect Answer

Equal  
Difficulty and Ability

Hover over parameters  
to see explanation

## Solving a problem (Slide Layer)



### Rasch Model (II)

**Probability Function:**

$$P_{nix} = \frac{\exp[x_{ni}(\theta_n - \delta_i)]}{\sum_{x_{ni}} \exp[x_{ni}(\theta_n - \delta_i)]}$$

**Response Probability:**

Correctly Answer      Incorrect Answer      Equal Difficulty and Ability

$$P_{ni1} = \frac{\exp(\theta_n - \delta_i)}{1 + \exp(\theta_n - \delta_i)}$$

Hover over parameters to see explanation 

## Not solving a problem (Slide Layer)



### Rasch Model (II)

**Probability Function:**

$$P_{nix} = \frac{\exp[x_{ni}(\theta_n - \delta_i)]}{\sum_{x_{ni}} \exp[x_{ni}(\theta_n - \delta_i)]}$$

**Response Probability:**

Correctly Answer      Incorrect Answer      Equal Difficulty and Ability

$$P_{ni0} = \frac{1}{1 + \exp(\theta_n - \delta_i)}$$

Hover over parameters to see explanation 

## Equal amounts (Slide Layer)

Rasch Model (II)

**Probability Function:**

$$P_{nix} = \frac{\exp[x_{ni}(\theta_n - \delta_i)]}{\sum_{x_i} \exp[x_{ni}(\theta_n - \delta_i)]}$$

**Response Probability:**

Correctly Answer

Incorrect Answer

Equal  
Difficulty and Ability

$$P_{ni1} = P_{ni0} = \frac{1}{2}$$

Hover over parameters  
to see explanation

## pnix (Slide Layer)

Rasch Model (II)

**Probability Function:**

$$P_{nix} = \frac{\exp[x_{ni}(\theta_n - \delta_i)]}{\sum_{x_i} \exp[x_{ni}(\theta_n - \delta_i)]}$$

Probability of person  $n$  answering  
item  $i$  with response  $x$   
(1 = correct, 0 = incorrect)

**Response Probability:**

Correctly Answer

Incorrect Answer

Equal  
Difficulty and Ability

Hover over parameters  
to see explanation

## xni (Slide Layer)

**Rasch Model (II)**

**Probability Function:**

$$P_{nix} = \frac{\exp(x_{ni}(\theta_n - \delta_i))}{\sum_{x_{ni}} \exp(x_{ni}(\theta_n - \delta_i))}$$

Scored response to item  $i$   
(1 = correct, 0 = incorrect)

**Response Probability:**

Correctly Answer      Incorrect Answer      Equal Difficulty and Ability

Hover over parameters to see explanation

## theta (Slide Layer)

**Rasch Model (II)**

**Probability Function:**

$$P_{nix} = \frac{\exp(x_{ni}(\theta_n - \delta_i))}{\sum_{x_{ni}} \exp(x_{ni}(\theta_n - \delta_i))}$$

Ability of person  $n$

**Response Probability:**

Correctly Answer      Incorrect Answer      Equal Difficulty and Ability

Hover over parameters to see explanation

## delta (Slide Layer)

  **Rasch Model (II)**

**Probability Function:**

$$P_{nix} = \frac{\exp(x_{ni}(\theta_n - \delta_i))}{\sum_{x_{ni}} \exp(x_{ni}(\theta_n - \delta_i))}$$

Difficulty of item  $i$

**Response Probability:**

Correctly Answer      Incorrect Answer      Equal Difficulty and Ability

Hover over parameters to see explanation



## exp (Slide Layer)

  **Rasch Model (II)**

**Probability Function:**

$$P_{nix} = \frac{\exp(x_{ni}(\theta_n - \delta_i))}{\sum_{x_{ni}} \exp(x_{ni}(\theta_n - \delta_i))}$$

Exponential function

**Response Probability:**

Correctly Answer      Incorrect Answer      Equal Difficulty and Ability

Hover over parameters to see explanation



## sum (Slide Layer)

🏠
Rasch Model (II)

**Probability Function:**

$$P_{nix} = \frac{\exp(x_{ni}(\theta_n))}{\sum_{x_{ni}} \exp(x_{ni}(\theta_n))}$$

Summation of probabilities  
(over all / both response categories)

**Response Probability:**

Correctly Answer

Incorrect Answer

Equal  
Difficulty and Ability

Hover over parameters  
to see explanation

## 3.15 Commonly Used Rasch Measurement Models

🏠
Common Rasch Models

Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\ln \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

## ability (Slide Layer)

Common Rasch Models

Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\ln \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

$\theta_n$  = ability of person  $n$

## difficulty (Slide Layer)

Common Rasch Models

Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\ln \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

$\delta_i$  = difficulty of item  $i$

## delta ik (Slide Layer)

Common Rasch Models

Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\ln \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

$\delta_{ik}$  = difficulty of step  $k$  of item  $i$  (assuming unique scale structure of each item)

## tau k (Slide Layer)

Common Rasch Models

Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\ln \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

$\tau_k$  = difficulty of step  $k$  (assuming common scale structure among all items)

## lamda m (Slide Layer)

Common Rasch Models

Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\ln \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

$\lambda_m =$  scoring severity of rater  $m$

## P ni1 (Slide Layer)

Common Rasch Models

Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\ln \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

$P_{ni1} =$  probability of a correct response to item  $i$  by person  $n$

## log (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\text{Ln} \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\text{Ln} \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\text{Ln} \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\text{Ln} \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\text{Ln} \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

Ln = logarithm

## P<sub>ni0</sub> (Slide Layer)

Common Rasch Models	
Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\text{Ln} \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\text{Ln} \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\text{Ln} \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\text{Ln} \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\text{Ln} \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

$P_{ni0}$  = probability of an incorrect response to item *i* by person *n*

## P nik (Slide Layer)

Common Rasch Models

Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\ln \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

$P_{nik}$  = probability of a response in category  $k$  of item  $i$  by person  $n$

## P nik-1 (Slide Layer)

Common Rasch Models

Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\ln \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

$P_{nik}$  = probability of a response in category  $(k-1)$  of item  $i$  by person  $n$

## P<sub>nmik</sub> (Slide Layer)

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Common Rasch Models

Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\ln \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

$P_{nmik}$  = probability of a response in category  $k$  of item  $i$  by rater  $m$  for person  $n$ 
👉

## P<sub>nmik-1</sub> (Slide Layer)

🏠
📊
Common Rasch Models

Model Name	Model Equation (Log-odds Formula)
<b>Single-facet Models</b>	
Dichotomous Model	$\ln \left( \frac{P_{ni1}}{P_{ni0}} \right) = \theta_n - \delta_i$
Partial Credit Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - \delta_{ik}$
Rating Scale Model	$\ln \left( \frac{P_{nik}}{P_{nik-1}} \right) = \theta_n - (\delta_i + \tau_k) = \theta_n - \delta_i - \tau_k$
<b>Many-facet Models</b>	
Many-facet Partial Credit Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - \delta_{ik}$
Many-facet Rating Scale Model	$\ln \left( \frac{P_{nmik}}{P_{nmik-1}} \right) = \theta_n - \lambda_m - (\delta_i + \tau_k) = \theta_n - \lambda_m - \delta_i - \tau_k$

$P_{nmik-1}$  = probability of a response in category  $(k-1)$  of item  $i$  by rater  $m$  for person  $n$ 
👉

### 3.16 Rasch Model Fits Inside the General 3PL Model

  **Rasch Model Fits Inside the General 3PL**

The Rasch Model is a special case of the more general 3-parameter logistic (3PL) model with  $c = 0.00$  and  $a = 1.00$ .

$$P_{ni1} = c_i + (1 + c_i) \frac{\exp[a_i(\theta_n - b_i)]}{1 + \exp[a_i(\theta_n - b_i)]}$$

$P_{ni1}$  = probability of correct response  
 $b_i$  = item difficulty parameter  
 $a_i$  = item discrimination parameter  
 $c_i$  = item lower asymptote (pseudo-guessing) parameter  
 $\theta_n$  = latent proficiency of person

- An Excel file for generating item response functions is in the Resources section of this player.

### 3.17 Bookend: Equations

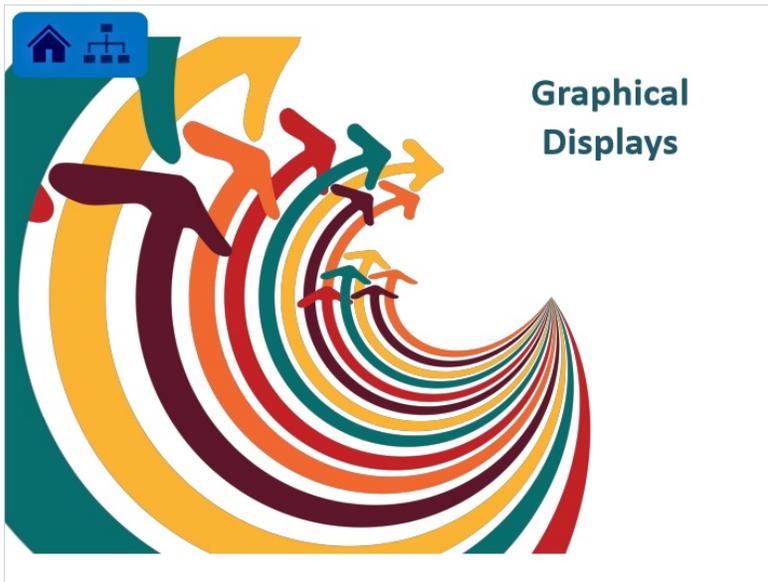
 



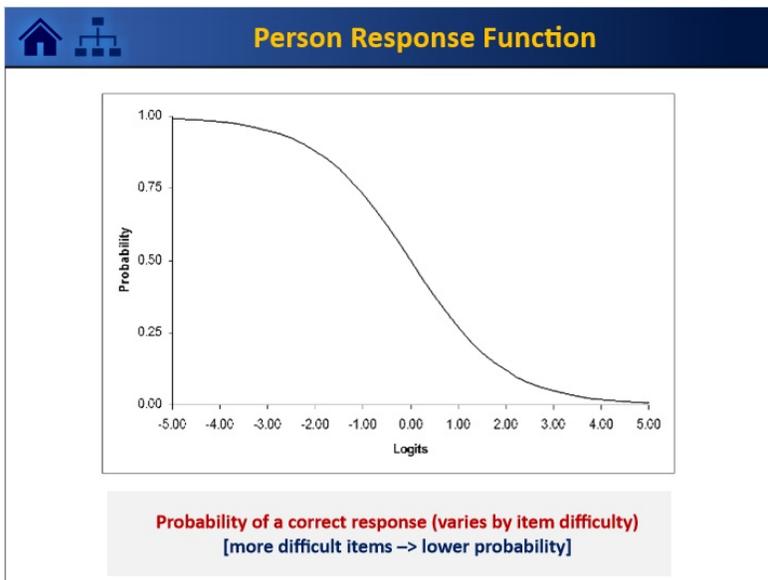
This is the end of this part.

**Topic Selection**

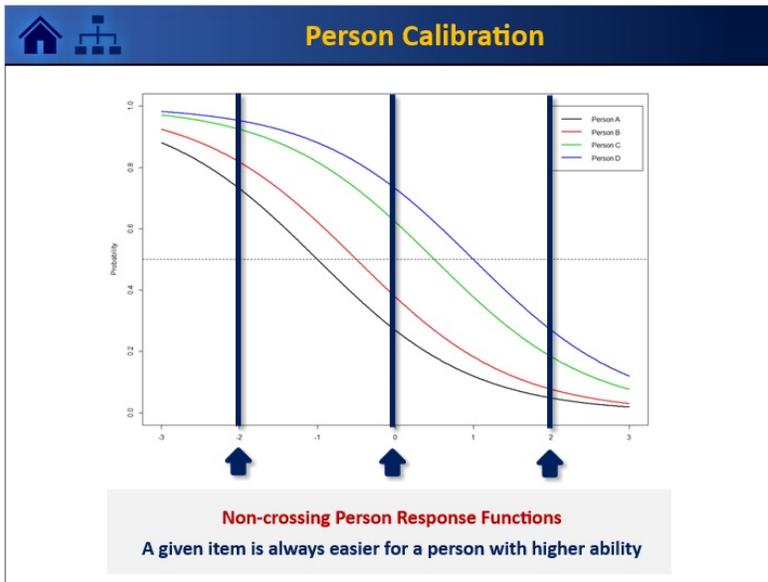
### 3.18 Bookmark: Graphs



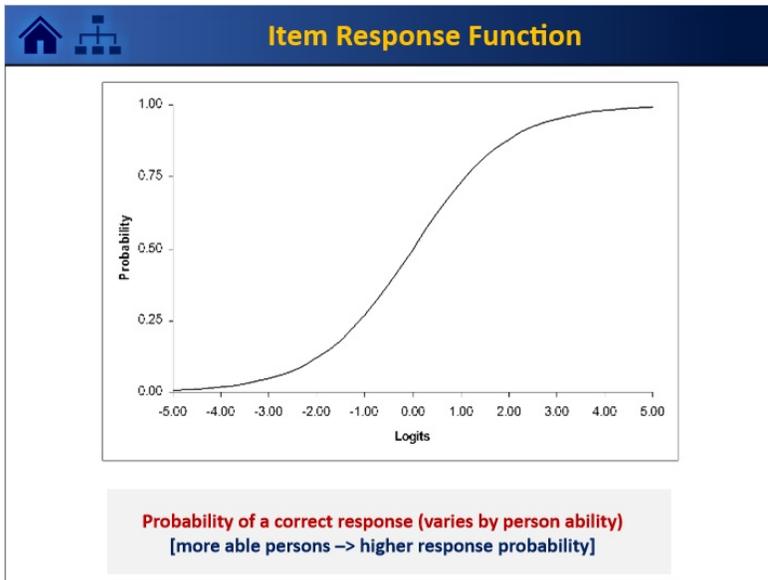
### 3.19 Person Response Function



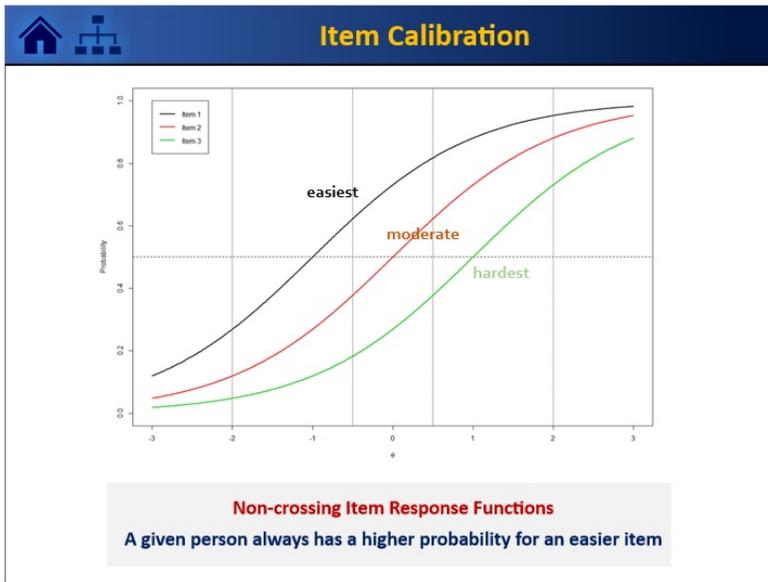
### 3.20 Person Measurement



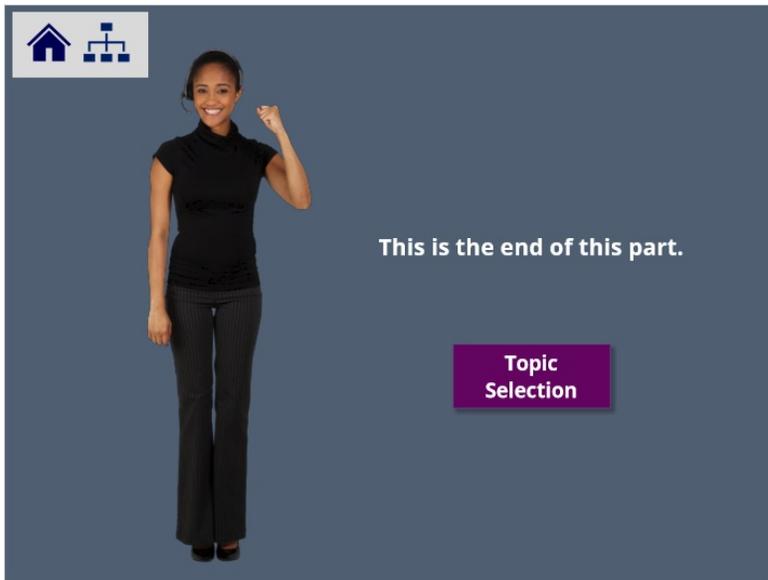
### 3.21 Item Response Function



### 3.22 Item Calibration



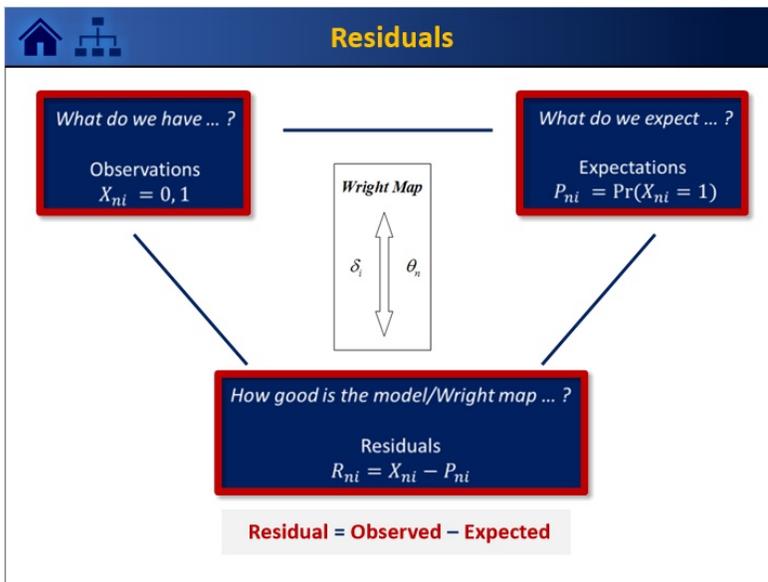
### 3.23 Bookend: Graphs



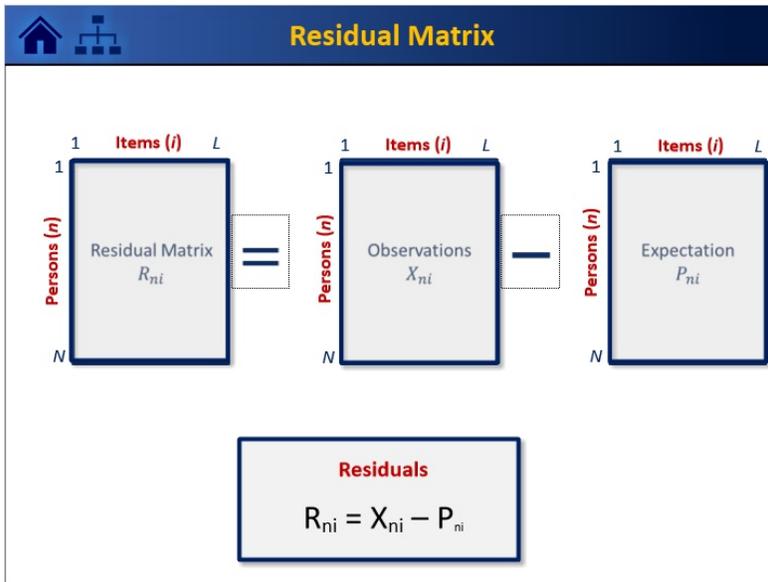
### 3.24 Bookmark: Fit



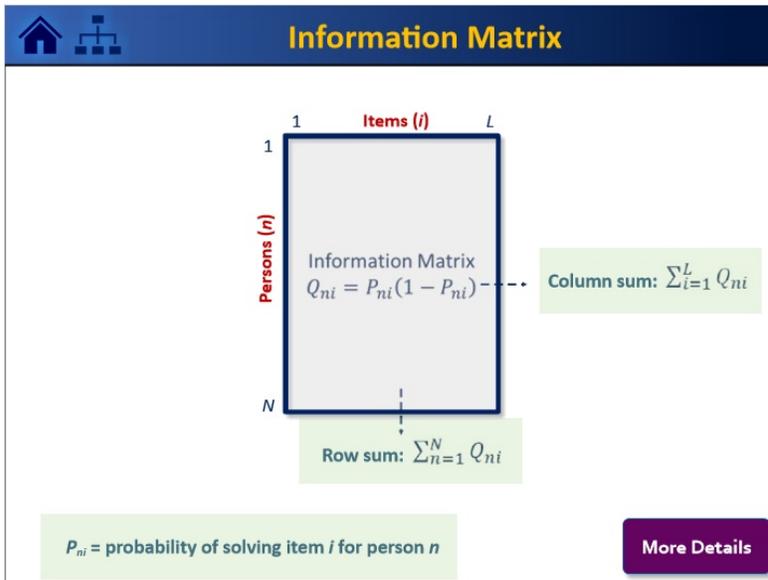
### 3.25 Residuals



### 3.26 Residual Matrix



### 3.27 Information Matrix



## References (Slide Layer)

🏠
Information Matrix



<https://www.rasch.org/rmt/rmt231j.htm>

Engelhard (2013, p.18)

Back

## 3.28 Mean Square Outfit

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Mean Square Outfit

	1	Items (i)	L
1			
⋮			
N			

$Z_{ni}^2 = \frac{R_{ni}^2}{Q_{ni}}$

- Expected value = 1
- Values close to 1 = good item / person fit
- Values away from 1 = misfit / investigation

Column sum:  $\sum_{i=1}^L Z_{ni}^2$

Person Outfit (column average):  
 $U_n = \frac{\sum_{i=1}^L Z_{ni}^2}{L}$

Residual:  $R_{ni} = X_{ni} - P_{ni}$   
 Information:  $Q_{ni} = P_{ni}(1 - P_{ni})$

Row sum:  $\sum_{n=1}^N Z_{ni}^2$

Item Outfit (row average):  
 $U_i = \frac{\sum_{n=1}^N Z_{ni}^2}{N}$

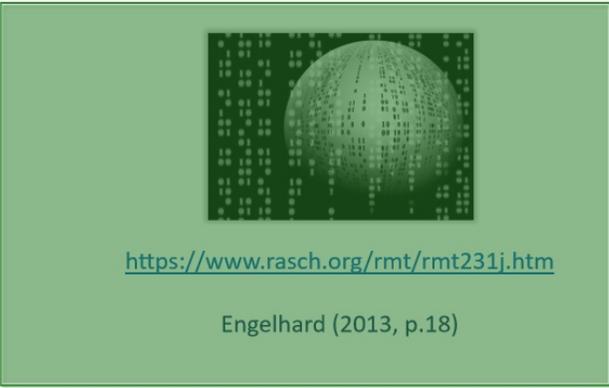
Reference



## References (Slide Layer)



### Mean Square Infit



<https://www.rasch.org/rmt/rmt231j.htm>

Engelhard (2013, p.18)

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## 3.30 Interpretation of Mean Square Statistics



### Interpretation of Mean Square Statistics

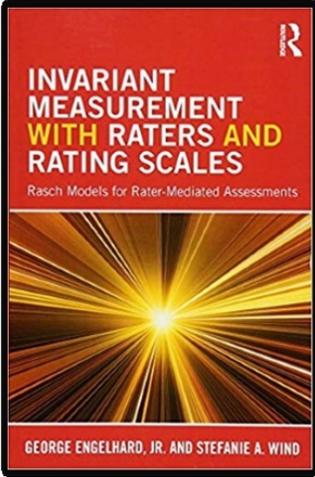
Mean Square Error (MSE)	Interpretation for Infit and Outfit	Fit Category
$.50 \leq MSE < 1.50$	Productive for measurement	A
$MSE < .50$	Less productive for measurement, but not distorting of measures	B
$1.50 \leq MSE < 2.00$	Unproductive for measurement, but not distorting of measures	C
$2.00 \leq MSE$	Unproductive for measurement, and distorting of measures	D

Note: These guidelines reflect rules of thumb.  
In practice, users should inspect the distributions of Infit and Outfit indicators.

Reference

## References (Slide Layer)

Reference



IN VARIANT  
MEASUREMENT  
WITH RATERS AND  
RATING SCALES  
Rasch Models for Rater-Mediated Assessments  
GEORGE ENGELHARD, JR. AND STEFANIE A. WIND

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### 3.31 Bookend: Fit



This is the end of this part.

Topic Selection

### 3.32 Summary (I)



#### Summary (I)



- **Rasch measurement models** are **confirmatory** in nature
- **Unidimensional** Rasch models assume a **single measurement continuum** for ability
- **Unidimensional** Rasch models capture only **one item characteristic** (e.g., difficulty) and **one person characteristic** (e.g., ability)
- A large **proportion of variance explained** indicates that a **common construct** can be well measured by the items
- **Unexplained variance** in Rasch models can be used to detect **additional dimensions** via **fit statistics**

### 3.33 Summary (II)



#### Summary (II)

**Rasch models provide an ordering of items and persons along a common continuum / on an underlying scale**

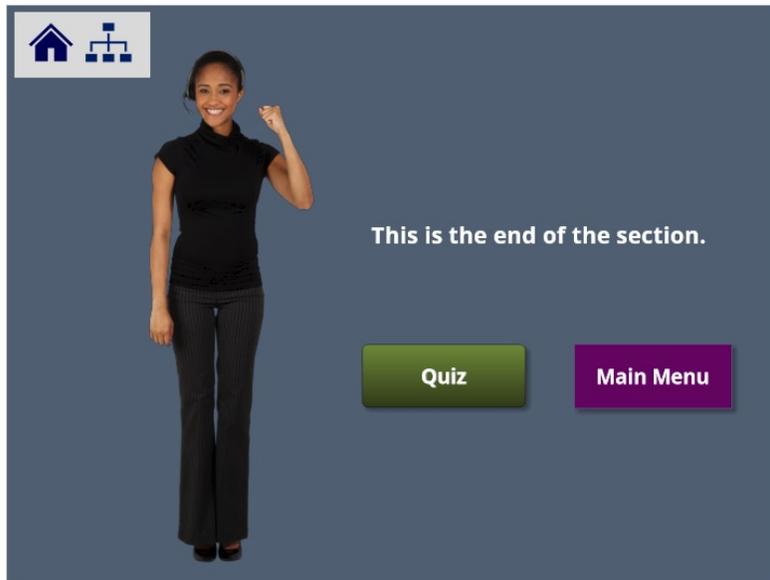
**When good fit of items and persons is achieved:**

- Ordering of items is invariant across person samples (e.g., **subpopulations**)
- Ordering of persons is invariant across item samples (e.g., **test forms**)

**Rasch models have non-crossing person and item response functions:**

- A more able person always has a better chance of success on an item than a less able person (**person response functions**)
- Any person always has a better chance of success on an easier item than on a more difficult item (**item response functions**)

### 3.34 Bookend: Section 2



## 4. Section 3: Creating a Rasch Scale

### 4.1 Cover: Section 3



## 4.2 Objectives: Section 3



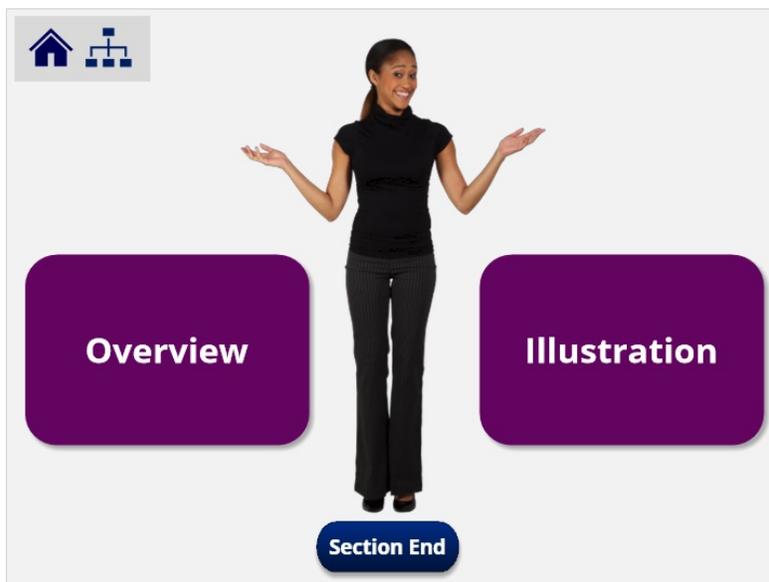
The slide features a dark blue header with a home icon and a tree icon on the left, and the text "Learning Objectives" in yellow on the right. Below the header is a target icon with a single arrow hitting the bullseye. Underneath the target are three white boxes with blue borders, each containing a numbered objective.

1. Explain the key steps for conceptualizing a construct

2. Describe the key steps for a Rasch model analysis

3. Interpret a Rasch scale for empirical examples

## 4.3 Topic Selection



The slide features a light gray background with a home icon and a tree icon in the top left corner. A woman in a black top and pants stands in the center with her arms outstretched. Below her are two large purple rounded rectangles labeled "Overview" and "Illustration". At the bottom center is a blue rounded rectangle labeled "Section End".

Overview

Illustration

Section End

## 4.4 Bookmark: Overview



## 4.5 Conceptualizing a Construct

 **Conceptualizing a Construct (!)**

**Latent variable**

- What is the construct?
- Is it unidimensional?
- Can person responses to items represent different levels of the construct?

---

**Observational design**

- What are the categories for the construct and item formats?
- What is the plan for collecting person responses?



## 4.6 Conceptualizing a Construct

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Conceptualizing a Construct

**Scoring rules**

- What is the interpretation of each category?
- How do we score the categories?

---

**Rasch measurement model**

- Can we map persons and items onto an invariant and unidimensional scale?
- How well do persons and items fit the Rasch measurement model?



## 4.7 Creating a Hypothesized Wright Map

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Creating a Hypothesized Wright Map

How do we use it?

**Step 1:** Define latent variable

**Step 2:** Map items along the logit scale based on hypothesized item locations

**Step 3:** Determine the response format



Open Checklist

What is the latent variable (construct)?

Logit Scale	Persons	Items
5.00		[Hard items]
4.00	High values on the latent variable	
3.00		
2.00		
1.00		
.00	Midrange values on the latent variable	[Moderately difficult items]
-1.00		
-2.00		
-3.00		
-4.00	Low values on the latent variable	
-5.00		[Easy items]

What is the response format or rating scale used?

- Dichotomous (x=0, 1)
- Polytomous (x=0, 1, 2, 3, ...)

## Checklist (Slide Layer)



### Checklist

- ✓ Make sure every item measures the latent variable
- ✓ Make sure items follow a cumulative scale
- ✓ Make sure items have different degrees of difficulty to target persons with varying locations along the latent variable



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## 4.8 Procedures for Rasch Model Analysis



### Procedures for Rasch Model Analysis

- Model**
  - **Dimensionality of scale** (unidimensional, multidimensional)
  - **Response format** (dichotomous, polytomous)
  - **Scale structures of items** (common, different)
  - **Number of facets** (e.g., item, person, and rater)
- Estimation**
  - **Maximum likelihood** (e.g., joint, conditional, marginal)
  - **Bayesian** (e.g., MCMC, prior specification, posterior inference)
  - **Other** (e.g., normal approximation, pairwise algorithm)
- Software**
  - **Specialized programs** (e.g., Winsteps, Facets, RUMM2030)
  - **General programs** (e.g., SAS, STATA, Mplus, R packages)



[www.rasch.org](http://www.rasch.org)

## 4.9 Interpreting a Unidimensional Rasch Scale



### Interpreting a Unidimensional Rasch Scale

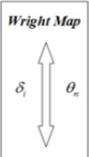
**Dimensionality analysis**

- Scree plot from exploratory factor analysis
- Proportion of variance explained



---

**Wright Map**



**Empirical Wright map**

- Relative ordering of items and persons
- Scale structures for polytomous items

---

**Item and person summary statistics**

- Item locations
- Person locations



## 4.10 Interpreting a Unidimensional Rasch Scale



### Interpreting a Unidimensional Rasch Scale

**Fit indices & reliability of separation**

- Person fit and person reliability
- Item fit and item reliability



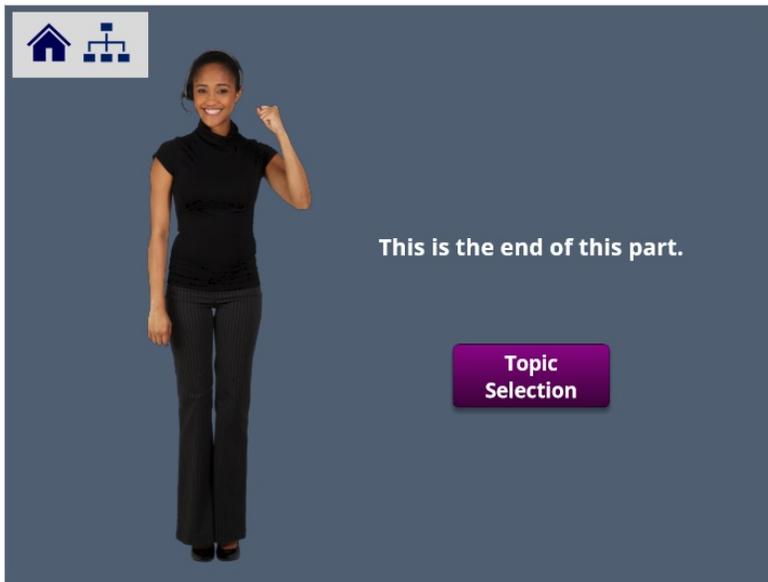
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**Precision**

- Standard error of measurement for persons
- Standard error of calibration for items



#### 4.11 Bookend: Overview



#### 4.12 Bookmark: Illustration



## 4.13 An Illustrative Example

Home
An Illustrative Example

Item	Description of Item
1	Child has toys which teach color, size, and shape.
2	Child has three or more puzzles.
3	Child has record player and at least five children's records.
4	Child has toys permitting free expression
5	Child has toys or games requiring refined movements.
6	Child has toys or games which help teach numbers.
7	Child has at least 10 children's books.
8	At least 10 books are visible in the apartment.
9	Family buys and reads a daily newspaper.
10	Family subscribes to at least one magazine.
11	Child is encouraged to learn shapes.

Reference

## Instrument (Slide Layer)

Home
Instrument

National Longitudinal Surveys

A Program of the U.S. Bureau of Labor Statistics

Search site

Getting Started
Cohorts
Access Data / Investigator
Request Questions for Future NLSY Surveys

[Home](#)
[Cohorts](#)
[NLSY79 Cohort](#)
[Special Studies in the Data](#)
[Assessments](#)

National Longitudinal Survey of Youth | 1979

Children and Young Adults

[NLSY79](#)
[NLSY79](#)
[NLSY79 Cohort](#)
[Method and Timing](#)
[Other and Young Men](#)

[Assessments](#)

- Intellectual
- The HOME
- Temperament
- Motor and Social Development
- Pediatric Checklist Index

- Early Mathematics
- Early Reading
- STICIS
- Self-Expression Profile for Children
- Motor Skills Data Sheet

**The HOME (Home Observation Measurement of the Environment)**

**Created variables**

HOMEAyyyy HOME INVENTORY PART A (0-2 YRS): TOTAL RAW SCORE

HOMEByyyy HOME INVENTORY PART B (3-5 YRS): TOTAL RAW SCORE

HOMEByyyy HOME INVENTORY PART C (6-8 YRS): TOTAL RAW SCORE

HOMEByyyy HOME INVENTORY PART D (9-14 YRS): TOTAL RAW SCORE

HOMEZyyy HOME INVENTORY: TOTAL STANDARD SCORE

HOMEYyyy HOME INVENTORY: TOTAL PERCENTILE SCORE

COGKAyyyy HOME INVENTORY PART A (0-2 YRS): COGNITIVE STIMULATION RAW SCORE

COGKByyyy HOME INVENTORY PART B (3-5 YRS): COGNITIVE STIMULATION RAW SCORE

COGKCyyyy HOME INVENTORY PART C (6-8 YRS): COGNITIVE STIMULATION RAW SCORE

COGKDyyyy HOME INVENTORY PART D (9-14 YRS): COGNITIVE STIMULATION RAW SCORE

COGKYyyy HOME INVENTORY: COGNITIVE STIMULATION STANDARD SCORE

COGZYyyy HOME INVENTORY: COGNITIVE STIMULATION PERCENTILE SCORE

EMO1Ayyyy HOME INVENTORY PART A (0-2 YRS): EMOTIONAL SUPPORT RAW SCORE

EMO1Byyyy HOME INVENTORY PART B (3-5 YRS): EMOTIONAL SUPPORT RAW SCORE

EMO1Cyyyy HOME INVENTORY PART C (6-8 YRS): EMOTIONAL SUPPORT RAW SCORE

EMO1Dyyyy HOME INVENTORY PART D (9-14 YRS): EMOTIONAL SUPPORT RAW SCORE

EMO1Zyyy HOME INVENTORY: EMOTIONAL SUPPORT STANDARD SCORE

EMO1Yyyy HOME INVENTORY: EMOTIONAL SUPPORT PERCENTILE SCORE

HOMEFyyyy HOME INVENTORY PART A (0-2 YRS): TOTAL RAW SCORE # ITEMS MISSING

HOMEFyyyy HOME INVENTORY PART B (3-5 YRS): TOTAL RAW SCORE # ITEMS MISSING

HOMEFyyyy HOME INVENTORY PART C (6-8 YRS): TOTAL RAW SCORE # ITEMS MISSING

HOMEFyyyy HOME INVENTORY PART D (9-14 YRS): TOTAL RAW SCORE # ITEMS MISSING (available for 1996-current survey waves)

Back

## 4.14 Building Blocks for Constructing Measures

Building Blocks	Questions	Answers	Learning Stimulation (LS) Scale
Latent variable	What is the latent variable being measured?	Learning Stimulation	The purpose of the LS scale is to measure the degree of learning stimulation in the home environment of preschool children.
Observational design	What is the plan for collecting structured responses or observations from persons?	Items: List of activities available in the home environment	Dichotomous ratings obtained from teachers visiting the homes of preschool children.
Scoring rules	How are responses or observations categorized to represent person levels on the latent variable?	Objects and activities scored not present ( $x = 0$ ) or present ( $x = 1$ )	Activities and objects checked as present indicate a higher level of learning stimulation in the home environment (0 = not present, 1 = present)
Measurement model	How are person and item responses or observations mapped onto the latent variable?	Rasch Model	Dichotomous Rasch Model

## 4.15 Data Description and Model Selection

Data and Model	
<b>Data Description</b>	
<ul style="list-style-type: none"> <li>Home environments of <b>40 preschool children</b> were evaluated based on <b>11 items</b> on the <b>LSS instrument</b></li> <li><b>Teachers</b> responded based on their <b>observations</b> during the <b>home visits</b></li> </ul>	
<b>Model Selection</b>	
• <b>Unidimensional construct:</b>	learning stimulation in home environments
• <b>Response format:</b>	binary (0 = absence, 1 = presence)
• <b>Measurement facets:</b>	
• Item difficulty:	difficulty to observe in a home environment
• Stimulation degree:	degree of enriching and stimulating learning opportunities in home environment
• <b>Model choice:</b>	Rasch model for dichotomous responses

## 4.16 Summary Table

Model-fit Summary		
Measures	Person	Item
Mean	.89	.00
Standard Deviation	1.55	1.76
N	40	11
<b>Infit MSE</b>		
Mean	1.00	.98
Standard Deviation	.41	.19
<b>Outfit MSE</b>		
Mean	.86	.86
Standard Deviation	.69	.42
<b>Reliability of Separation</b>	.69	.92
<b>% of Variance Explained:</b>	44.6%	

Item Details
Person Details

## Item Breakdown (Slide Layer)

Item Breakdown								
Item No.	Description	Proportion	Measure	S.E.	OUTFIT		INFIT	
					MSE	Fit Category	MSE	Fit Category
1	Child has toys which teach color, size, and shape.	0.88	-1.72	0.57	0.84	A	1.01	A
2	Child has three or more puzzles.	0.48	1.34	0.39	1.07	A	0.87	A
3	Child has record player and at least five children's records.	0.38	1.98	0.41	1.06	A	1.18	A
4	Child has toys permitting free expression	0.93	-2.49	0.68	0.20	B	0.67	A
5	Child has toys or games requiring refined movements.	0.60	0.57	0.40	0.75	A	0.94	A
6	Child has toys or games which help teach numbers.	0.83	-1.14	0.51	1.13	A	1.21	A
7	Child has at least 10 children's books.	0.68	0.08	0.41	0.62	A	0.78	A
8	At least 10 books are visible in the apartment.	0.63	0.42	0.40	1.04	A	0.94	A
9	Family buys and reads a daily newspaper.	0.43	1.65	0.40	1.75	C	1.37	A
10	Family subscribes to at least one magazine.	0.33	2.33	0.43	0.80	A	0.95	A
11	shapes.	0.95	-3.03	0.79	0.22	B	0.87	A

Classification Summary
Back to Main Slide

## Person Breakdown (Slide Layer)

Person Breakdown

Person No.	Proportion	Measure	S.E.	OUTFIT		INFIT	
				MSE	Fit Category	MSE	Fit Category

Click on each row to see a residual plot for that person.

Back to Main Slide

## Person 33 (Slide Layer)

Person 33: Overfit

Standardized Residual (Person 33)

Items

Back to Person Table

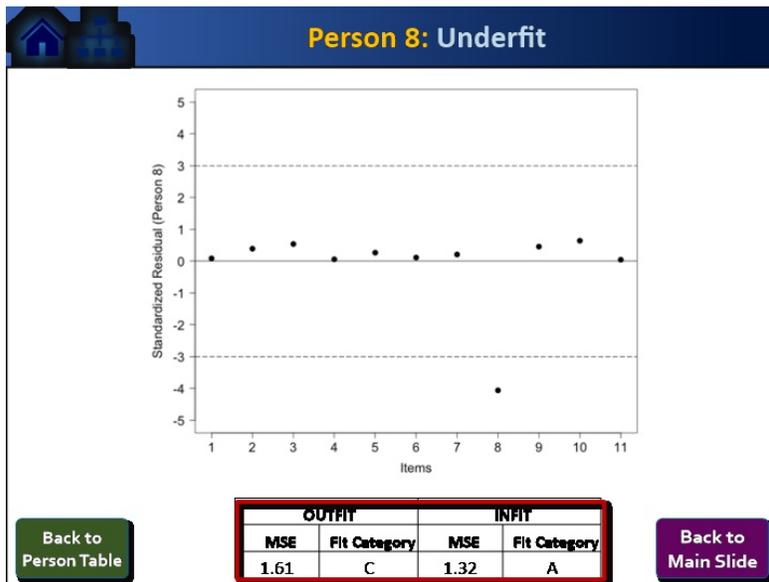
OUTFIT		INFIT	
MSE	Fit Category	MSE	Fit Category
0.23	B	0.47	B

Back to Main Slide

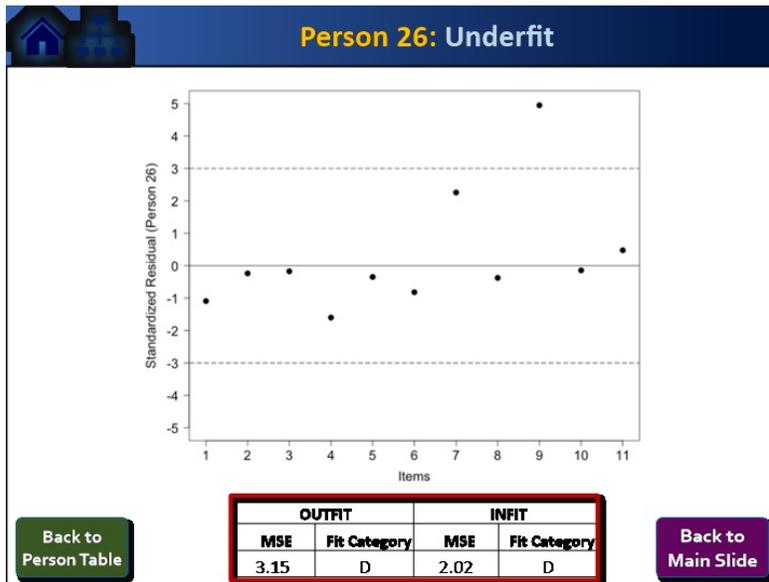
## Person 35 (Slide Layer)



## Person 8 (Slide Layer)



## Person 26 (Slide Layer)



## Classification (Slide Layer)

**Item Classification**

Index	Fit Category			
	A	B	C	D
Person Oufit	.48	.35	.05	.12
Person Infit	.78	.05	.12	.05
Item Oufit	.73	.18	.09	.00
Item Infit	1.00	.00	.00	.00

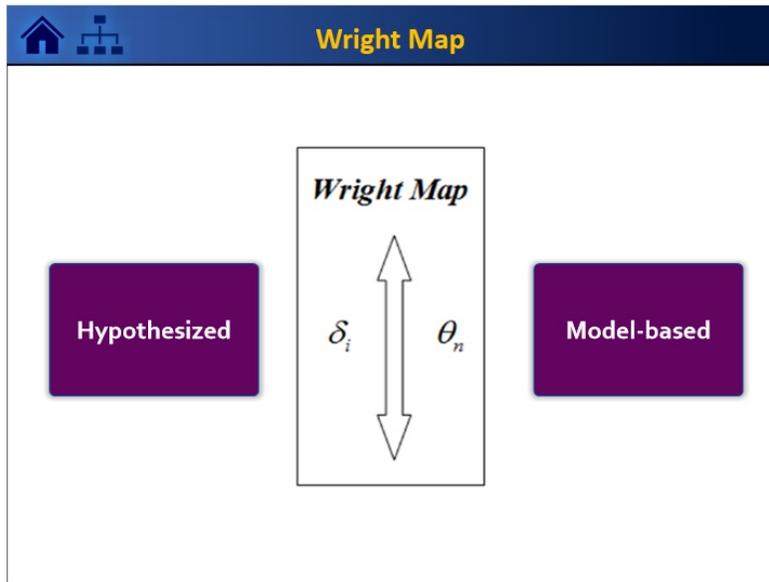
Proportion of cases in each category

Mean Square Error (MSE)	Interpretation	Fit Category
$.50 \leq \text{MSE} < 1.50$	Productive for measurement	A
$\text{MSE} < .50$	Less productive for measurement, but not distorting of measures	B
$1.50 \leq \text{MSE} < 2.00$	Unproductive for measurement, but not distorting of measures	C
$2.00 \leq \text{MSE}$	Unproductive for measurement, and distorting of measures	D

General interpretation guidelines

Buttons: [Back to Item Table](#) [Back to Main Slide](#)

## 4.17 Untitled Slide



## Hypothesized (Slide Layer)

The slide is titled "Hypothesized" and contains a table with two columns: "Home Environments of Preschool Children" and "Home Observations". The table is a 5-point Likert scale with response options from 5 (High) to 1 (Low). Below the table, there is a note about the response format and a "Back" button.

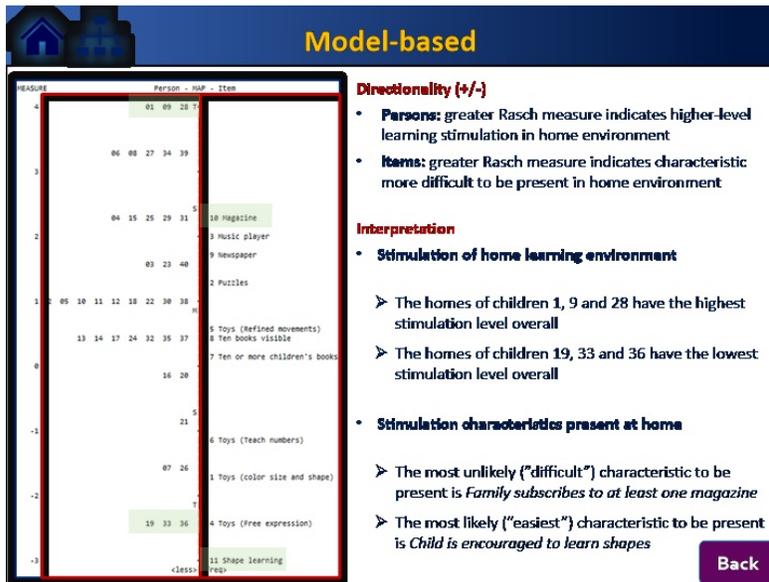
Home Environments of Preschool Children	Home Observations
5 (High)	5 (High)
4 (High)	4 (High)
3 (Midrange)	3 (Midrange)
2 (Midrange)	2 (Midrange)
1 (Low)	1 (Low)
5 (High)	5 (High)
4 (High)	4 (High)
3 (Midrange)	3 (Midrange)
2 (Midrange)	2 (Midrange)
1 (Low)	1 (Low)
5 (High)	5 (High)
4 (High)	4 (High)
3 (Midrange)	3 (Midrange)
2 (Midrange)	2 (Midrange)
1 (Low)	1 (Low)

What is the response format or rating scale used?  
 • Dichotomous scores are used with *not present* coded as zero (x=0) and *present* coded as one (x=1)

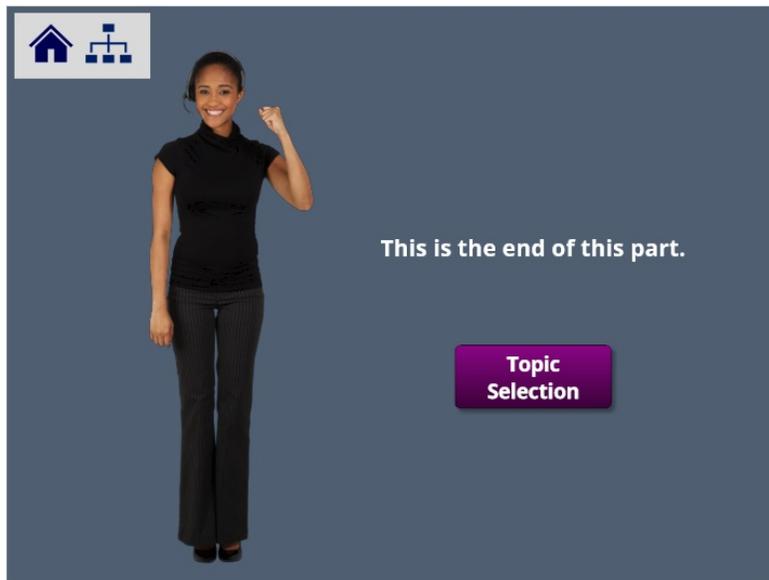
Source: Engelhard (2013, p. 7)

Back

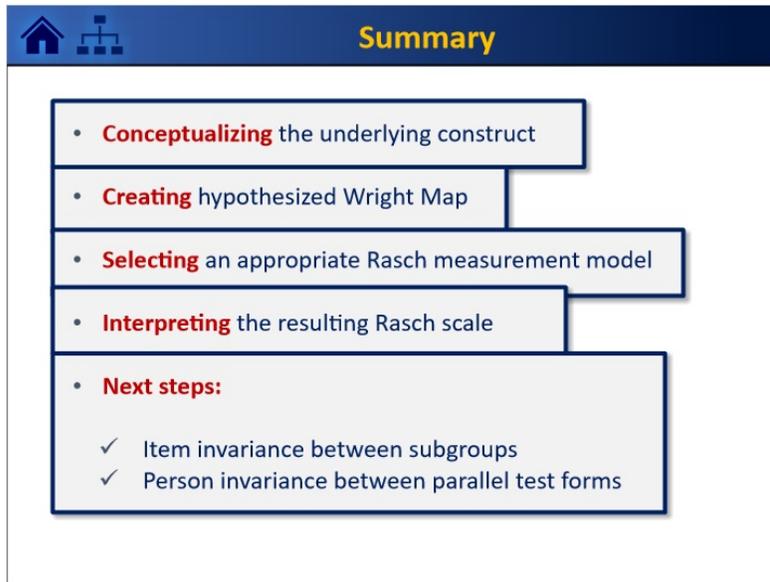
## Model-based (Slide Layer)



## 4.18 Bookend: Illustration



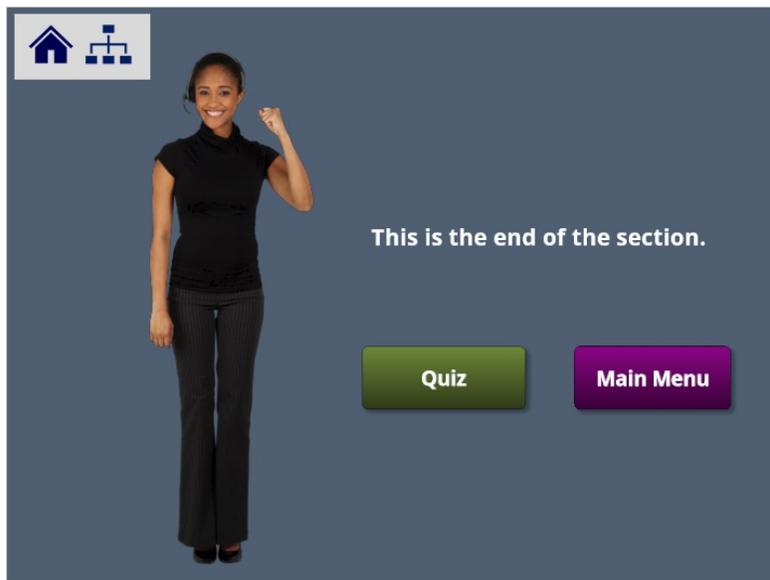
## 4.19 Summary



**Summary**

- **Conceptualizing** the underlying construct
- **Creating** hypothesized Wright Map
- **Selecting** an appropriate Rasch measurement model
- **Interpreting** the resulting Rasch scale
- **Next steps:**
  - ✓ Item invariance between subgroups
  - ✓ Person invariance between parallel test forms

## 4.20 Bookend: Section 3



**This is the end of the section.**

[Quiz](#) [Main Menu](#)

## 4.21 Module Cover (END)

**NCME**  
**ITEMS Module**  
Rasch Measurement Theory

PDF Document

PDF Document

END OF SECTION 3

Instructors Get Started Designers